

Maple 2018.2 Integration Test Results  
on the problems in "6 Hyperbolic functions/6.7 Miscellaneous"

Test results for the 276 problems in "6.7.1 Hyperbolic functions.txt"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(2+3x)^2}{1+2 \tanh(2+3x)^2} dx$$

Optimal(type 3, 16 leaves, 2 steps):

$$\frac{\arctan(\sqrt{2} \tanh(2+3x)) \sqrt{2}}{6}$$

Result(type 3, 155 leaves):

$$\frac{\sqrt{6} \arctan\left(\frac{2 \tanh\left(1 + \frac{3x}{2}\right)}{2\sqrt{3} - 2\sqrt{2}}\right)}{3(2\sqrt{3} - 2\sqrt{2})} - \frac{2 \arctan\left(\frac{2 \tanh\left(1 + \frac{3x}{2}\right)}{2\sqrt{3} - 2\sqrt{2}}\right)}{3(2\sqrt{3} - 2\sqrt{2})} - \frac{\sqrt{6} \arctan\left(\frac{2 \tanh\left(1 + \frac{3x}{2}\right)}{2\sqrt{3} + 2\sqrt{2}}\right)}{3(2\sqrt{3} + 2\sqrt{2})} - \frac{2 \arctan\left(\frac{2 \tanh\left(1 + \frac{3x}{2}\right)}{2\sqrt{3} + 2\sqrt{2}}\right)}{3(2\sqrt{3} + 2\sqrt{2})}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{csch}(2+3x)^2}{1-2 \coth(2+3x)^2} dx$$

Optimal(type 3, 17 leaves, 2 steps):

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2} \tanh(2+3x)}{2}\right) \sqrt{2}}{6}$$

Result(type 3, 101 leaves):

$$-\frac{\sqrt{2} \ln\left(\frac{\tanh\left(1 + \frac{3x}{2}\right)^2 + \tanh\left(1 + \frac{3x}{2}\right) \sqrt{2} + 1}{\tanh\left(1 + \frac{3x}{2}\right)^2 - \tanh\left(1 + \frac{3x}{2}\right) \sqrt{2} + 1}\right)}{24} + \frac{\sqrt{2} \ln\left(\frac{\tanh\left(1 + \frac{3x}{2}\right)^2 - \tanh\left(1 + \frac{3x}{2}\right) \sqrt{2} + 1}{\tanh\left(1 + \frac{3x}{2}\right)^2 + \tanh\left(1 + \frac{3x}{2}\right) \sqrt{2} + 1}\right)}{24}$$

Problem 4: Unable to integrate problem.

$$\int \cosh(bx+a)^3 \sinh(bx+a)^n dx$$

Optimal(type 3, 39 leaves, 3 steps):

$$\frac{\sinh(bx+a)^{1+n}}{b(1+n)} + \frac{\sinh(bx+a)^{3+n}}{b(3+n)}$$

Result(type 8, 19 leaves):

$$\int \cosh(bx + a)^3 \sinh(bx + a)^n dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{\sinh(bx + a)^{7/2}}{\cosh(bx + a)^{7/2}} dx$$

Optimal(type 3, 88 leaves, 6 steps):

$$-\frac{\arctan\left(\frac{\sqrt{\cosh(bx + a)}}{\sqrt{\sinh(bx + a)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\cosh(bx + a)}}{\sqrt{\sinh(bx + a)}}\right)}{b} - \frac{2 \sinh(bx + a)^{5/2}}{5b \cosh(bx + a)^{5/2}} - \frac{2\sqrt{\sinh(bx + a)}}{b\sqrt{\cosh(bx + a)}}$$

Result(type 8, 19 leaves):

$$\int \frac{\sinh(bx + a)^{7/2}}{\cosh(bx + a)^{7/2}} dx$$

Problem 18: Unable to integrate problem.

$$\int \frac{\cosh(bx + a)^{7/2}}{\sinh(bx + a)^{7/2}} dx$$

Optimal(type 3, 88 leaves, 6 steps):

$$-\frac{\arctan\left(\frac{\sqrt{\sinh(bx + a)}}{\sqrt{\cosh(bx + a)}}\right)}{b} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\sinh(bx + a)}}{\sqrt{\cosh(bx + a)}}\right)}{b} - \frac{2 \cosh(bx + a)^{5/2}}{5b \sinh(bx + a)^{5/2}} - \frac{2\sqrt{\cosh(bx + a)}}{b\sqrt{\sinh(bx + a)}}$$

Result(type 8, 19 leaves):

$$\int \frac{\cosh(bx + a)^{7/2}}{\sinh(bx + a)^{7/2}} dx$$

Problem 19: Unable to integrate problem.

$$\int \frac{\sinh(bx + a)^{4/3}}{\cosh(bx + a)^{4/3}} dx$$

Optimal(type 3, 197 leaves, 12 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\cosh(bx + a)^{1/3}}{\sinh(bx + a)^{1/3}}\right)}{b} - \frac{\ln\left(1 + \frac{\cosh(bx + a)^{2/3}}{\sinh(bx + a)^{2/3}} - \frac{\cosh(bx + a)^{1/3}}{\sinh(bx + a)^{1/3}}\right)}{4b} + \frac{\ln\left(1 + \frac{\cosh(bx + a)^{2/3}}{\sinh(bx + a)^{2/3}} + \frac{\cosh(bx + a)^{1/3}}{\sinh(bx + a)^{1/3}}\right)}{4b}$$

$$-\frac{3 \sinh(bx+a)^{1/3}}{b \cosh(bx+a)^{1/3}} + \frac{\arctan\left(\frac{\left(1 - \frac{2 \cosh(bx+a)^{1/3}}{\sinh(bx+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b} - \frac{\arctan\left(\frac{\left(1 + \frac{2 \cosh(bx+a)^{1/3}}{\sinh(bx+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b}$$

Result(type 8, 19 leaves):

$$\int \frac{\sinh(bx+a)^{4/3}}{\cosh(bx+a)^{4/3}} dx$$

Problem 20: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)^{4/3}}{\sinh(bx+a)^{4/3}} dx$$

Optimal(type 3, 197 leaves, 12 steps):

$$\frac{\operatorname{arctanh}\left(\frac{\sinh(bx+a)^{1/3}}{\cosh(bx+a)^{1/3}}\right)}{b} - \frac{\ln\left(1 - \frac{\sinh(bx+a)^{1/3}}{\cosh(bx+a)^{1/3}} + \frac{\sinh(bx+a)^{2/3}}{\cosh(bx+a)^{2/3}}\right)}{4b} + \frac{\ln\left(1 + \frac{\sinh(bx+a)^{1/3}}{\cosh(bx+a)^{1/3}} + \frac{\sinh(bx+a)^{2/3}}{\cosh(bx+a)^{2/3}}\right)}{4b}$$

$$-\frac{3 \cosh(bx+a)^{1/3}}{b \sinh(bx+a)^{1/3}} + \frac{\arctan\left(\frac{\left(1 - \frac{2 \sinh(bx+a)^{1/3}}{\cosh(bx+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b} - \frac{\arctan\left(\frac{\left(1 + \frac{2 \sinh(bx+a)^{1/3}}{\cosh(bx+a)^{1/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b}$$

Result(type 8, 19 leaves):

$$\int \frac{\cosh(bx+a)^{4/3}}{\sinh(bx+a)^{4/3}} dx$$

Problem 21: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)^{5/3}}{\sinh(bx+a)^{5/3}} dx$$

Optimal(type 3, 124 leaves, 9 steps):

$$-\frac{\ln\left(1 - \frac{\sinh(bx+a)^{2/3}}{\cosh(bx+a)^{2/3}}\right)}{2b} + \frac{\ln\left(1 + \frac{\sinh(bx+a)^{2/3}}{\cosh(bx+a)^{2/3}} + \frac{\sinh(bx+a)^{4/3}}{\cosh(bx+a)^{4/3}}\right)}{4b} - \frac{3 \cosh(bx+a)^{2/3}}{2b \sinh(bx+a)^{2/3}}$$

$$-\frac{\arctan\left(\frac{\left(1 + \frac{2 \sinh(bx+a)^{2/3}}{\cosh(bx+a)^{2/3}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b}$$

Result(type 8, 19 leaves):

$$\int \frac{\cosh(bx+a)^{5/\sqrt{3}}}{\sinh(bx+a)^{5/\sqrt{3}}} dx$$

Problem 22: Unable to integrate problem.

$$\int \frac{\cosh(bx+a)^{7/\sqrt{3}}}{\sinh(bx+a)^{7/\sqrt{3}}} dx$$

Optimal(type 3, 124 leaves, 9 steps):

$$\begin{aligned} & -\frac{\ln\left(1 - \frac{\cosh(bx+a)^{2/\sqrt{3}}}{\sinh(bx+a)^{2/\sqrt{3}}}\right)}{2b} + \frac{\ln\left(1 + \frac{\cosh(bx+a)^{4/\sqrt{3}}}{\sinh(bx+a)^{4/\sqrt{3}}} + \frac{\cosh(bx+a)^{2/\sqrt{3}}}{\sinh(bx+a)^{2/\sqrt{3}}}\right)}{4b} - \frac{3 \cosh(bx+a)^{4/\sqrt{3}}}{4b \sinh(bx+a)^{4/\sqrt{3}}} \\ & - \frac{\arctan\left(\frac{\left(1 + \frac{2 \cosh(bx+a)^{2/\sqrt{3}}}{\sinh(bx+a)^{2/\sqrt{3}}}\right)\sqrt{3}}{3}\right)\sqrt{3}}{2b} \end{aligned}$$

Result(type 8, 19 leaves):

$$\int \frac{\cosh(bx+a)^{7/\sqrt{3}}}{\sinh(bx+a)^{7/\sqrt{3}}} dx$$

Problem 26: Unable to integrate problem.

$$\int \operatorname{sech}(bx+a)^4 \sqrt{\tanh(bx+a)} dx$$

Optimal(type 3, 27 leaves, 3 steps):

$$\frac{2 \tanh(bx+a)^{3/2}}{3b} - \frac{2 \tanh(bx+a)^{7/2}}{7b}$$

Result(type 8, 19 leaves):

$$\int \operatorname{sech}(bx+a)^4 \sqrt{\tanh(bx+a)} dx$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \operatorname{sech}(bx+a)^4 \tanh(bx+a)^n dx$$

Optimal(type 3, 40 leaves, 3 steps):

$$\frac{\tanh(bx+a)^{1+n}}{b(1+n)} - \frac{\tanh(bx+a)^{3+n}}{b(3+n)}$$

Result(type 3, 534 leaves):

$$\frac{1}{b(1+n)(3+n)(1+e^{2bx+2a})^3} \left( 2(e^{6bx+6a} + 2ne^{4bx+4a} + 3e^{4bx+4a} - 2ne^{2bx+2a} - 3e^{2bx+2a} - 1) \right. \\ \left. - \frac{1}{2} \left( n \left( \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^3 \pi - \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^2 \operatorname{csgn}(I(e^{bx+a}+1)) \pi - \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right) \operatorname{csgn}\left(\frac{I}{1+e^{2bx+2a}}\right) \pi \right. \right. \right. \\ \left. \left. + \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right) \operatorname{csgn}(I(e^{bx+a}+1)) \operatorname{csgn}\left(\frac{I}{1+e^{2bx+2a}}\right) \pi - \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right) \operatorname{csgn}\left(\frac{I(e^{bx+a}-1)(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^2 \pi \right. \right. \\ \left. \left. + \operatorname{Icsgn}\left(\frac{I(e^{bx+a}+1)}{1+e^{2bx+2a}}\right) \operatorname{csgn}\left(\frac{I(e^{bx+a}-1)(e^{bx+a}+1)}{1+e^{2bx+2a}}\right) \operatorname{csgn}(I(e^{bx+a}-1)) \pi + \operatorname{Icsgn}\left(\frac{I(e^{bx+a}-1)(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^3 \pi - \operatorname{Icsgn}\left(\frac{I(e^{bx+a}-1)(e^{bx+a}+1)}{1+e^{2bx+2a}}\right)^2 \operatorname{csgn}(I(e^{bx+a} \right. \right. \\ \left. \left. - 1)) \pi + 2 \ln(1+e^{2bx+2a}) - 2 \ln(e^{bx+a}-1) - 2 \ln(e^{bx+a}+1) \right) \right) \right)$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \operatorname{sech}(x)^8 \tanh(x)^6 dx$$

Optimal(type 3, 25 leaves, 3 steps):

$$\frac{\tanh(x)^7}{7} - \frac{\tanh(x)^9}{3} + \frac{3 \tanh(x)^{11}}{11} - \frac{\tanh(x)^{13}}{13}$$

Result(type 3, 71 leaves):

$$-\frac{\sinh(x)^5}{8 \cosh(x)^{13}} - \frac{\sinh(x)^3}{16 \cosh(x)^{13}} - \frac{\sinh(x)}{64 \cosh(x)^{13}} \\ + \frac{\left( \frac{1024}{3003} + \frac{\operatorname{sech}(x)^{12}}{13} + \frac{12 \operatorname{sech}(x)^{10}}{143} + \frac{40 \operatorname{sech}(x)^8}{429} + \frac{320 \operatorname{sech}(x)^6}{3003} + \frac{128 \operatorname{sech}(x)^4}{1001} + \frac{512 \operatorname{sech}(x)^2}{3003} \right) \tanh(x)}{64}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \operatorname{coth}(bx+a)^3 \operatorname{csch}(bx+a)^3 dx$$

Optimal(type 3, 27 leaves, 3 steps):

$$-\frac{\operatorname{csch}(bx+a)^3}{3b} - \frac{\operatorname{csch}(bx+a)^5}{5b}$$

Result(type 3, 67 leaves):

$$\frac{-\frac{\cosh(bx+a)^2}{5 \sinh(bx+a)^5} - \frac{2 \cosh(bx+a)^2}{15 \sinh(bx+a)^3} + \frac{2 \cosh(bx+a)^2}{15 \sinh(bx+a)} - \frac{2 \sinh(bx+a)}{15}}{b}$$

Problem 35: Result more than twice size of optimal antiderivative.

$$\int \coth(bx+a)^3 \operatorname{csch}(bx+a)^n dx$$

Optimal(type 3, 37 leaves, 3 steps):

$$-\frac{\operatorname{csch}(bx+a)^n}{bn} - \frac{\operatorname{csch}(bx+a)^{2+n}}{b(2+n)}$$

Result(type 3, 498 leaves):

$$-\frac{1}{bn(2+n)(e^{2bx+2a}-1)^2} \left( (ne^{4bx+4a} + 2e^{4bx+4a} + 2ne^{2bx+2a} - 4e^{2bx+2a} + n \right. \\ \left. + 2) \right. \\ \left. e^{-\frac{1}{2}} \left( n \left( \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \right)^3 \pi - \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \right)^2 \operatorname{csgn}\left(\frac{1}{e^{bx+a}-1}\right) \pi - \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \right)^2 \operatorname{csgn}\left(\frac{1}{e^{bx+a}+1}\right) \pi \right. \\ \left. + \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \operatorname{csgn}\left(\frac{1}{e^{bx+a}-1}\right) \operatorname{csgn}\left(\frac{1}{e^{bx+a}+1}\right) \pi - \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \operatorname{csgn}\left(\frac{1e^{bx+a}}{(e^{bx+a}+1)(e^{bx+a}-1)}\right) \right)^2 \pi \right. \\ \left. + \operatorname{Icsgn}\left(\frac{1}{(e^{bx+a}-1)(e^{bx+a}+1)}\right) \operatorname{csgn}\left(\frac{1e^{bx+a}}{(e^{bx+a}+1)(e^{bx+a}-1)}\right) \operatorname{csgn}(1e^{bx+a}) \pi + \operatorname{Icsgn}\left(\frac{1e^{bx+a}}{(e^{bx+a}+1)(e^{bx+a}-1)}\right) \right)^3 \pi \right. \\ \left. - \operatorname{Icsgn}\left(\frac{1e^{bx+a}}{(e^{bx+a}+1)(e^{bx+a}-1)}\right) \right)^2 \operatorname{csgn}(1e^{bx+a}) \pi + 2 \ln(e^{bx+a}-1) - 2 \ln(e^{bx+a}) + 2 \ln(e^{bx+a}+1) - 2 \ln(2) \left. \right) \left. \right)$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \coth(x)^2 \operatorname{csch}(x)^4 dx$$

Optimal(type 3, 13 leaves, 3 steps):

$$\frac{\coth(x)^3}{3} - \frac{\coth(x)^5}{5}$$

Result(type 3, 27 leaves):

$$-\frac{\cosh(x)}{4 \sinh(x)^5} - \frac{\left(-\frac{8}{15} - \frac{\operatorname{csch}(x)^4}{5} + \frac{4 \operatorname{csch}(x)^2}{15}\right) \operatorname{coth}(x)}{4}$$

Problem 38: Result more than twice size of optimal antiderivative.

$$\int \operatorname{coth}(x)^n \operatorname{csch}(x)^4 dx$$

Optimal(type 3, 26 leaves, 3 steps):

$$\frac{\operatorname{coth}(x)^{1+n}}{1+n} - \frac{\operatorname{coth}(x)^{3+n}}{3+n}$$

Result(type 3, 370 leaves):

$$-\frac{1}{(1+n)(3+n)(e^{2x}-1)^3} \left( 2(-e^{6x} + 2ne^{4x} + 3e^{4x} + 2ne^{2x} + 3e^{2x} - 1) \right. \\ \left. - \frac{1}{e} \left( n \left( I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right) \right)^3 - I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right)^2 \operatorname{csgn}\left(\frac{I}{1+e^x}\right) - I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right)^2 \operatorname{csgn}(I(e^{2x}+1)) + I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right) \operatorname{csgn}\left(\frac{I}{1+e^x}\right) \operatorname{csgn}(I(e^{2x}+1)) \right. \right. \\ \left. - I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right) \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{(e^x-1)(1+e^x)}\right)^2 + I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{1+e^x}\right) \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{(e^x-1)(1+e^x)}\right) \operatorname{csgn}\left(\frac{I}{e^x-1}\right) + I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{(e^x-1)(1+e^x)}\right)^3 \right. \\ \left. \left. - I\pi \operatorname{csgn}\left(\frac{I(e^{2x}+1)}{(e^x-1)(1+e^x)}\right)^2 \operatorname{csgn}\left(\frac{I}{e^x-1}\right) + 2 \ln(e^x-1) - 2 \ln(e^{2x}+1) + 2 \ln(1+e^x) \right) \right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int -\operatorname{csch}(bx-c) \operatorname{csch}(bx+a) dx$$

Optimal(type 3, 36 leaves, 3 steps):

$$-\frac{\operatorname{csch}(a+c) \ln(-\sinh(bx-c))}{b} + \frac{\operatorname{csch}(a+c) \ln(\sinh(bx+a))}{b}$$

Result(type 3, 76 leaves):

$$-\frac{2 \ln(-e^{2a+2c} + e^{2bx+2a}) e^{a+c}}{(e^{2a+2c}-1)b} + \frac{2 \ln(e^{2bx+2a}-1) e^{a+c}}{(e^{2a+2c}-1)b}$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \coth(bx+c)^2 \sinh(bx+a) dx$$

Optimal(type 3, 46 leaves, 6 steps):

$$-\frac{\operatorname{arctanh}(\cosh(bx+c)) \cosh(a-c)}{b} + \frac{\cosh(bx+a)}{b} - \frac{\operatorname{csch}(bx+c) \sinh(a-c)}{b}$$

Result(type 3, 196 leaves):

$$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}(e^{2c}-e^{2a})}{b(e^{2bx+2a+2c}-e^{2a})} + \frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2c}}{2b} + \frac{\ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2a}}{2b} - \frac{\ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2c}}{2b} - \frac{\ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2a}}{2b}$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \coth(bx+c)^3 \sinh(bx+a) dx$$

Optimal(type 3, 69 leaves, 9 steps):

$$-\frac{\cosh(a-c) \operatorname{csch}(bx+c)}{b} - \frac{3 \operatorname{arctanh}(\cosh(bx+c)) \sinh(a-c)}{2b} - \frac{\coth(bx+c) \operatorname{csch}(bx+c) \sinh(a-c)}{2b} + \frac{\sinh(bx+a)}{b}$$

Result(type 3, 229 leaves):

$$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} - \frac{e^{bx+a}(e^{2bx+2a+4c}+3e^{2bx+4a+2c}-3e^{2a+2c}-e^{4a})}{2b(e^{2bx+2a+2c}-e^{2a})^2} - \frac{3 \ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2c}}{4b} + \frac{3 \ln(e^{bx+a}-e^{a-c})e^{-a-c}e^{2a}}{4b} + \frac{3 \ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2c}}{4b} - \frac{3 \ln(e^{bx+a}+e^{a-c})e^{-a-c}e^{2a}}{4b}$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \operatorname{csch}(bx+c) \sinh(bx+a) dx$$

Optimal(type 3, 26 leaves, 3 steps):

$$x \cosh(a-c) + \frac{\ln(\sinh(bx+c)) \sinh(a-c)}{b}$$

Result(type 3, 149 leaves):

$$xe^{a-c} + e^{-a-c}e^{2c}x - e^{-a-c}e^{2a}x + \frac{e^{-a-c}e^{2c}a}{b} - \frac{e^{-a-c}e^{2a}a}{b} - \frac{\ln(e^{2bx+2a}-e^{2a-2c})e^{-a-c}e^{2c}}{2b} + \frac{\ln(e^{2bx+2a}-e^{2a-2c})e^{-a-c}e^{2a}}{2b}$$

Problem 45: Result more than twice size of optimal antiderivative.

$$\int \cosh(bx+a) \tanh(bx+c) dx$$

Optimal(type 3, 29 leaves, 3 steps):



$$\frac{\cosh(bx+a)}{b} - \frac{\arctan(\sinh(bx+c)) \sinh(a-c)}{b}$$

Result(type 3, 166 leaves):

$$\frac{e^{bx+a}}{2b} + \frac{e^{-bx-a}}{2b} + \frac{\text{Iln}(e^{bx+a} + Ie^{a-c}) e^{-a-c} (e^c)^2}{2b} - \frac{\text{Iln}(e^{bx+a} + Ie^{a-c}) e^{-a-c} (e^a)^2}{2b} - \frac{\text{Iln}(e^{bx+a} - Ie^{a-c}) e^{-a-c} (e^c)^2}{2b} + \frac{\text{Iln}(e^{bx+a} - Ie^{a-c}) e^{-a-c} (e^a)^2}{2b}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \cosh(bx+a) \tanh(bx+c)^2 dx$$

Optimal(type 3, 45 leaves, 6 steps):

$$-\frac{\arctan(\sinh(bx+c)) \cosh(a-c)}{b} + \frac{\text{sech}(bx+c) \sinh(a-c)}{b} + \frac{\sinh(bx+a)}{b}$$

Result(type 3, 207 leaves):

$$\frac{e^{bx+a}}{2b} - \frac{e^{-bx-a}}{2b} - \frac{e^{bx+a} (e^{2c} - e^{2a})}{b (e^{2bx+2a+2c} + e^{2a})} + \frac{\text{Iln}(e^{bx+a} - Ie^{a-c}) e^{-a-c} e^{2c}}{2b} + \frac{\text{Iln}(e^{bx+a} - Ie^{a-c}) e^{-a-c} e^{2a}}{2b} - \frac{\text{Iln}(e^{bx+a} + Ie^{a-c}) e^{-a-c} e^{2c}}{2b} - \frac{\text{Iln}(e^{bx+a} + Ie^{a-c}) e^{-a-c} e^{2a}}{2b}$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \cosh(bx+a) \text{csch}(bx+c)^2 dx$$

Optimal(type 3, 36 leaves, 4 steps):

$$-\frac{\cosh(a-c) \text{csch}(bx+c)}{b} - \frac{\text{arctanh}(\cosh(bx+c)) \sinh(a-c)}{b}$$

Result(type 3, 170 leaves):

$$-\frac{e^{bx+a} (e^{2c} + e^{2a})}{b (e^{2bx+2a+2c} - e^{2a})} - \frac{\ln(e^{bx+a} - e^{a-c}) e^{-a-c} e^{2c}}{2b} + \frac{\ln(e^{bx+a} - e^{a-c}) e^{-a-c} e^{2a}}{2b} + \frac{\ln(e^{bx+a} + e^{a-c}) e^{-a-c} e^{2c}}{2b} - \frac{\ln(e^{bx+a} + e^{a-c}) e^{-a-c} e^{2a}}{2b}$$

Problem 49: Unable to integrate problem.

$$\int \sinh(bx+a) \tanh(dx+c) dx$$

Optimal(type 5, 109 leaves, 6 steps):

$$\frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}}{2b} - \frac{e^{-bx-a} \text{hypergeom}\left(\left[1, -\frac{b}{2d}\right], \left[1 - \frac{b}{2d}\right], -e^{2dx+2c}\right)}{b} - \frac{e^{bx+a} \text{hypergeom}\left(\left[1, \frac{b}{2d}\right], \left[1 + \frac{b}{2d}\right], -e^{2dx+2c}\right)}{b}$$

Result(type 8, 59 leaves):

$$\frac{e^{bx+a}}{2b} + \frac{1}{2be^{bx+a}} + \int -\frac{(e^{bx+a})^2 - 1}{e^{bx+a}((e^{dx+c})^2 + 1)} dx$$

Problem 50: Unable to integrate problem.

$$\int \cosh(bx+a) \coth(dx+c) dx$$

Optimal(type 5, 104 leaves, 6 steps):

$$-\frac{e^{-bx-a}}{2b} + \frac{e^{bx+a}}{2b} + \frac{e^{-bx-a} \operatorname{hypergeom}\left(\left[1, -\frac{b}{2d}\right], \left[1 - \frac{b}{2d}\right], e^{2dx+2c}\right)}{b} - \frac{e^{bx+a} \operatorname{hypergeom}\left(\left[1, \frac{b}{2d}\right], \left[1 + \frac{b}{2d}\right], e^{2dx+2c}\right)}{b}$$

Result(type 8, 58 leaves):

$$\frac{e^{bx+a}}{2b} - \frac{1}{2be^{bx+a}} + \int \frac{(e^{bx+a})^2 + 1}{((e^{dx+c})^2 - 1)e^{bx+a}} dx$$

Problem 52: Result more than twice size of optimal antiderivative.

$$\int \sinh(x) \tanh(2x) dx$$

Optimal(type 3, 15 leaves, 4 steps):

$$\sinh(x) - \frac{\arctan(\sinh(x)\sqrt{2})\sqrt{2}}{2}$$

Result(type 3, 53 leaves):

$$\frac{e^x}{2} - \frac{e^{-x}}{2} + \frac{I\sqrt{2} \ln(e^{2x} - I\sqrt{2}e^x - 1)}{4} - \frac{I\sqrt{2} \ln(e^{2x} + I\sqrt{2}e^x - 1)}{4}$$

Problem 53: Unable to integrate problem.

$$\int \sinh(x) \tanh(nx) dx$$

Optimal(type 5, 67 leaves, 6 steps):

$$\frac{1}{2e^x} + \frac{e^x}{2} - \frac{\operatorname{hypergeom}\left(\left[1, -\frac{1}{2n}\right], \left[1 - \frac{1}{2n}\right], -e^{2nx}\right)}{e^x} - e^x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], -e^{2nx}\right)$$

Result(type 8, 35 leaves):

$$\frac{e^x}{2} + \frac{1}{2e^x} + \int -\frac{(e^x)^2 - 1}{e^x((e^{nx})^2 + 1)} dx$$

Problem 54: Result more than twice size of optimal antiderivative.

$$\int \coth(3x) \sinh(x) dx$$

Optimal(type 3, 16 leaves, 3 steps):

$$\sinh(x) - \frac{\arctan\left(\frac{2 \sinh(x) \sqrt{3}}{3}\right) \sqrt{3}}{3}$$

Result(type 3, 50 leaves):

$$-\frac{1}{1 + \tanh\left(\frac{x}{2}\right)} - \frac{\sqrt{3} \arctan\left(\frac{\tanh\left(\frac{x}{2}\right) \sqrt{3}}{3}\right)}{3} - \frac{1}{\tanh\left(\frac{x}{2}\right) - 1} - \frac{\sqrt{3} \arctan\left(\tanh\left(\frac{x}{2}\right) \sqrt{3}\right)}{3}$$

Problem 55: Result more than twice size of optimal antiderivative.

$$\int \coth(4x) \sinh(x) dx$$

Optimal(type 3, 20 leaves, 6 steps):

$$-\frac{\arctan(\sinh(x))}{4} + \sinh(x) - \frac{\arctan(\sinh(x) \sqrt{2}) \sqrt{2}}{4}$$

Result(type 3, 142 leaves):

$$-\frac{1}{1 + \tanh\left(\frac{x}{2}\right)} - \frac{1}{\tanh\left(\frac{x}{2}\right) - 1} - \frac{\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2} + \frac{\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{2(-2 + 2\sqrt{2})} - \frac{\arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{-2 + 2\sqrt{2}} - \frac{\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2(2 + 2\sqrt{2})} - \frac{\arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2 + 2\sqrt{2}}$$

Problem 56: Result more than twice size of optimal antiderivative.

$$\int \operatorname{csch}(3x) \sinh(x) dx$$

Optimal(type 3, 13 leaves, 2 steps):

$$\frac{\arctan\left(\frac{\tanh(x) \sqrt{3}}{3}\right) \sqrt{3}}{3}$$

Result(type 3, 39 leaves):

$$\frac{I\sqrt{3} \ln\left(e^{2x} + \frac{1}{2} + \frac{I\sqrt{3}}{2}\right)}{6} - \frac{I\sqrt{3} \ln\left(e^{2x} + \frac{1}{2} - \frac{I\sqrt{3}}{2}\right)}{6}$$

Problem 57: Result more than twice size of optimal antiderivative.

$$\int \operatorname{csch}(6x) \sinh(x) \, dx$$

Optimal(type 3, 26 leaves, 7 steps):

$$\frac{\arctan(\sinh(x))}{6} + \frac{\arctan(2 \sinh(x))}{6} - \frac{\arctan\left(\frac{2 \sinh(x) \sqrt{3}}{3}\right) \sqrt{3}}{6}$$

Result(type 3, 91 leaves):

$$\frac{I \ln(e^x + I)}{6} - \frac{I \ln(e^x - I)}{6} + \frac{I\sqrt{3} \ln(e^{2x} - I\sqrt{3} e^x - 1)}{12} - \frac{I\sqrt{3} \ln(e^{2x} + I\sqrt{3} e^x - 1)}{12} + \frac{I \ln(e^{2x} + I e^x - 1)}{12} - \frac{I \ln(e^{2x} - I e^x - 1)}{12}$$

Problem 64: Result more than twice size of optimal antiderivative.

$$\int \cosh(x) \coth(6x) \, dx$$

Optimal(type 3, 28 leaves, 7 steps):

$$-\frac{\operatorname{arctanh}(\cosh(x))}{6} - \frac{\operatorname{arctanh}(2 \cosh(x))}{6} + \cosh(x) - \frac{\operatorname{arctanh}\left(\frac{2 \cosh(x) \sqrt{3}}{3}\right) \sqrt{3}}{6}$$

Result(type 3, 86 leaves):

$$\frac{e^x}{2} + \frac{e^{-x}}{2} - \frac{\ln(1 + e^x)}{6} + \frac{\ln(e^x - 1)}{6} + \frac{\sqrt{3} \ln(e^{2x} - \sqrt{3} e^x + 1)}{12} - \frac{\sqrt{3} \ln(e^{2x} + \sqrt{3} e^x + 1)}{12} + \frac{\ln(e^{2x} - e^x + 1)}{12} - \frac{\ln(e^{2x} + e^x + 1)}{12}$$

Problem 65: Unable to integrate problem.

$$\int \cosh(x) \coth(nx) \, dx$$

Optimal(type 5, 62 leaves, 6 steps):

$$-\frac{1}{2e^x} + \frac{e^x}{2} + \frac{\operatorname{hypergeom}\left(\left[1, -\frac{1}{2n}\right], \left[1 - \frac{1}{2n}\right], e^{2nx}\right)}{e^x} - e^x \operatorname{hypergeom}\left(\left[1, \frac{1}{2n}\right], \left[1 + \frac{1}{2n}\right], e^{2nx}\right)$$

Result(type 8, 34 leaves):

$$\frac{e^x}{2} - \frac{1}{2e^x} + \int \frac{(e^x)^2 + 1}{((e^{nx})^2 - 1) e^x} \, dx$$

Problem 66: Result is not expressed in closed-form.

$$\int \cosh(x) \operatorname{sech}(4x) dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$\frac{\arctan\left(\frac{2 \sinh(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{4-2\sqrt{2}}} - \frac{\arctan\left(\frac{2 \sinh(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{4+2\sqrt{2}}}$$

Result(type 7, 39 leaves):

$$2 \left( \sum_{R=\operatorname{RootOf}(32768 Z^4+512 Z^2+1)} {}_R \ln(e^{2x} + (-4096 R^3 - 48 R) e^x - 1) \right)$$

Problem 67: Result is not expressed in closed-form.

$$\int \cosh(x) \operatorname{sech}(5x) dx$$

Optimal(type 3, 49 leaves, 4 steps):

$$-\frac{\arctan\left(\sqrt{5-2\sqrt{5}} \tanh(x)\right) \sqrt{10-2\sqrt{5}}}{10} + \frac{\arctan\left(\sqrt{5+2\sqrt{5}} \tanh(x)\right) \sqrt{10+2\sqrt{5}}}{10}$$

Result(type 7, 40 leaves):

$$2 \left( \sum_{R=\operatorname{RootOf}(32000 Z^4+400 Z^2+1)} {}_R \ln(-4000 R^3 + 200 R^2 + e^{2x} - 30 R + 1) \right)$$

Problem 68: Result more than twice size of optimal antiderivative.

$$\int \cosh(x) \operatorname{csch}(4x) dx$$

Optimal(type 3, 18 leaves, 4 steps):

$$-\frac{\operatorname{arctanh}(\cosh(x))}{4} + \frac{\operatorname{arctanh}(\cosh(x) \sqrt{2}) \sqrt{2}}{4}$$

Result(type 3, 52 leaves):

$$-\frac{\ln(1+e^x)}{4} + \frac{\ln(e^x-1)}{4} + \frac{\ln(1+e^{2x}+e^x\sqrt{2})\sqrt{2}}{8} - \frac{\ln(1+e^{2x}-e^x\sqrt{2})\sqrt{2}}{8}$$

Problem 69: Unable to integrate problem.

$$\int x^m \cosh(bx+a) \sinh(bx+a) dx$$

Optimal(type 4, 70 leaves, 5 steps):

$$\frac{2^{-3-m} e^{2a} x^m \Gamma(1+m, -2bx)}{b(-bx)^m} + \frac{2^{-3-m} x^m \Gamma(1+m, 2bx)}{b e^{2a} (bx)^m}$$

Result(type 8, 18 leaves):

$$\int x^m \cosh(bx+a) \sinh(bx+a) dx$$

Problem 71: Unable to integrate problem.

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a) dx$$

Optimal(type 4, 124 leaves, 8 steps):

$$\frac{3^{-1-m} e^{3a} x^m \Gamma(1+m, -3bx)}{8b(-bx)^m} + \frac{e^a x^m \Gamma(1+m, -bx)}{8b(-bx)^m} + \frac{x^m \Gamma(1+m, bx)}{8b e^a (bx)^m} + \frac{3^{-1-m} x^m \Gamma(1+m, 3bx)}{8b e^{3a} (bx)^m}$$

Result(type 8, 20 leaves):

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a) dx$$

Problem 75: Result more than twice size of optimal antiderivative.

$$\int x^3 \cosh(bx+a) \sinh(bx+a)^2 dx$$

Optimal(type 3, 103 leaves, 7 steps):

$$\frac{14 \cosh(bx+a)}{9b^4} + \frac{2x^2 \cosh(bx+a)}{3b^2} - \frac{2 \cosh(bx+a)^3}{27b^4} - \frac{4x \sinh(bx+a)}{3b^3} - \frac{x^2 \cosh(bx+a) \sinh(bx+a)^2}{3b^2} + \frac{2x \sinh(bx+a)^3}{9b^3} + \frac{x^3 \sinh(bx+a)^3}{3b}$$

Result(type 3, 333 leaves):

$$\frac{1}{b^4} \left( \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{(bx+a)^3 \sinh(bx+a)}{3} - \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)}{3} + \frac{2(bx+a)^2 \cosh(bx+a)}{3} \right. \\ \left. + \frac{2(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{9} - \frac{14(bx+a) \sinh(bx+a)}{9} - \frac{2 \cosh(bx+a) \sinh(bx+a)^2}{27} + \frac{40 \cosh(bx+a)}{27} \right. \\ \left. - 3a \left( \frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{(bx+a)^2 \sinh(bx+a)}{3} - \frac{2(bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{9} + \frac{4(bx+a) \cosh(bx+a)}{9} \right. \right. \\ \left. \left. + \frac{2 \cosh(bx+a)^2 \sinh(bx+a)}{27} - \frac{14 \sinh(bx+a)}{27} \right) + 3a^2 \left( \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{(bx+a) \sinh(bx+a)}{3} \right. \right. \\ \left. \left. - \frac{\cosh(bx+a) \sinh(bx+a)^2}{9} + \frac{2 \cosh(bx+a)}{9} \right) - a^3 \left( \frac{\cosh(bx+a)^2 \sinh(bx+a)}{3} - \frac{\sinh(bx+a)}{3} \right) \right)$$

Problem 76: Result more than twice size of optimal antiderivative.

$$\int x^2 \cosh(bx+a) \sinh(bx+a)^2 dx$$

Optimal(type 3, 73 leaves, 4 steps):

$$\frac{4x \cosh(bx+a)}{9b^2} - \frac{4 \sinh(bx+a)}{9b^3} - \frac{2x \cosh(bx+a) \sinh(bx+a)^2}{9b^2} + \frac{2 \sinh(bx+a)^3}{27b^3} + \frac{x^2 \sinh(bx+a)^3}{3b}$$

Result(type 3, 192 leaves):

$$\frac{1}{b^3} \left( \frac{(bx+a)^2 \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{(bx+a)^2 \sinh(bx+a)}{3} - \frac{2(bx+a) \sinh(bx+a)^2 \cosh(bx+a)}{9} + \frac{4(bx+a) \cosh(bx+a)}{9} \right. \\ \left. + \frac{2 \cosh(bx+a)^2 \sinh(bx+a)}{27} - \frac{14 \sinh(bx+a)}{27} - 2a \left( \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^2}{3} - \frac{(bx+a) \sinh(bx+a)}{3} \right) \right. \\ \left. - \frac{\cosh(bx+a) \sinh(bx+a)^2}{9} + \frac{2 \cosh(bx+a)}{9} \right) + a^2 \left( \frac{\cosh(bx+a)^2 \sinh(bx+a)}{3} - \frac{\sinh(bx+a)}{3} \right)$$

Problem 78: Unable to integrate problem.

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^2 dx$$

Optimal(type 4, 87 leaves, 5 steps):

$$-\frac{x^{1+m}}{8(1+m)} + \frac{e^{4a} x^m \Gamma(1+m, -4bx)}{2^{6+2m} b (-bx)^m} - \frac{x^m \Gamma(1+m, 4bx)}{2^{6+2m} b e^{4a} (bx)^m}$$

Result(type 8, 22 leaves):

$$\int x^m \cosh(bx+a)^2 \sinh(bx+a)^2 dx$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int x^3 \cosh(bx+a)^3 \sinh(bx+a)^2 dx$$

Optimal(type 3, 178 leaves, 14 steps):

$$\frac{3 \cosh(bx+a)}{4b^4} + \frac{3x^2 \cosh(bx+a)}{8b^2} - \frac{\cosh(3bx+3a)}{216b^4} - \frac{x^2 \cosh(3bx+3a)}{48b^2} - \frac{3 \cosh(5bx+5a)}{5000b^4} - \frac{3x^2 \cosh(5bx+5a)}{400b^2} - \frac{3x \sinh(bx+a)}{4b^3} \\ - \frac{x^3 \sinh(bx+a)}{8b} + \frac{x \sinh(3bx+3a)}{72b^3} + \frac{x^3 \sinh(3bx+3a)}{48b} + \frac{3x \sinh(5bx+5a)}{1000b^3} + \frac{x^3 \sinh(5bx+5a)}{80b}$$

Result(type 3, 533 leaves):

$$\frac{1}{b^4} \left( \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^4}{5} - \frac{2(bx+a)^3 \sinh(bx+a)}{15} - \frac{(bx+a)^3 \sinh(bx+a) \cosh(bx+a)^2}{15} \right. \\ \left. - \frac{3(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)^3}{25} - \frac{4(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)}{75} + \frac{26(bx+a)^2 \cosh(bx+a)}{75} \right)$$

$$\begin{aligned}
& + \frac{6(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{125} - \frac{856(bx+a)\sinh(bx+a)}{1125} + \frac{22(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{1125} \\
& - \frac{6\cosh(bx+a)^3\sinh(bx+a)^2}{625} - \frac{272\cosh(bx+a)\sinh(bx+a)^2}{16875} + \frac{12568\cosh(bx+a)}{16875} - 3a \left( \frac{(bx+a)^2\sinh(bx+a)\cosh(bx+a)^4}{5} \right. \\
& - \frac{2(bx+a)^2\sinh(bx+a)}{15} - \frac{(bx+a)^2\sinh(bx+a)\cosh(bx+a)^2}{15} - \frac{2(bx+a)\sinh(bx+a)^2\cosh(bx+a)^3}{25} \\
& - \frac{8(bx+a)\sinh(bx+a)^2\cosh(bx+a)}{225} + \frac{52(bx+a)\cosh(bx+a)}{225} + \frac{2\cosh(bx+a)^4\sinh(bx+a)}{125} - \frac{856\sinh(bx+a)}{3375} \\
& \left. + \frac{22\cosh(bx+a)^2\sinh(bx+a)}{3375} \right) + 3a^2 \left( \frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^4}{5} - \frac{2(bx+a)\sinh(bx+a)}{15} \right. \\
& - \frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{15} - \frac{\cosh(bx+a)^3\sinh(bx+a)^2}{25} - \frac{4\cosh(bx+a)\sinh(bx+a)^2}{225} + \frac{26\cosh(bx+a)}{225} \left. \right) \\
& - a^3 \left( \frac{\cosh(bx+a)^4\sinh(bx+a)}{5} - \left( \frac{2}{3} + \frac{\cosh(bx+a)^2}{3} \right) \frac{\sinh(bx+a)}{5} \right)
\end{aligned}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int x^2 \cosh(bx+a)\sinh(bx+a)^3 dx$$

Optimal (type 3, 89 leaves, 4 steps):

$$-\frac{3x^2}{32b} + \frac{3x\cosh(bx+a)\sinh(bx+a)}{16b^2} - \frac{3\sinh(bx+a)^2}{32b^3} - \frac{x\cosh(bx+a)\sinh(bx+a)^3}{8b^2} + \frac{\sinh(bx+a)^4}{32b^3} + \frac{x^2\sinh(bx+a)^4}{4b}$$

Result (type 3, 236 leaves):

$$\begin{aligned}
& \frac{1}{b^3} \left( \frac{(bx+a)^2\sinh(bx+a)^2\cosh(bx+a)^2}{4} - \frac{(bx+a)^2\cosh(bx+a)^2}{4} - \frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{8} \right. \\
& + \frac{5(bx+a)\cosh(bx+a)\sinh(bx+a)}{16} + \frac{5(bx+a)^2}{32} + \frac{\cosh(bx+a)^2\sinh(bx+a)^2}{32} - \frac{\cosh(bx+a)^2}{8} \\
& - 2a \left( \frac{(bx+a)\sinh(bx+a)^2\cosh(bx+a)^2}{4} - \frac{\cosh(bx+a)^2(bx+a)}{4} - \frac{\sinh(bx+a)\cosh(bx+a)^3}{16} + \frac{5\cosh(bx+a)\sinh(bx+a)}{32} + \frac{5bx}{32} \right. \\
& \left. \left. + \frac{5a}{32} \right) + a^2 \left( \frac{\cosh(bx+a)^2\sinh(bx+a)^2}{4} - \frac{\cosh(bx+a)^2}{4} \right)
\end{aligned}$$



Problem 84: Unable to integrate problem.

$$\int x^m \cosh(bx + a)^2 \sinh(bx + a)^3 dx$$

Optimal(type 4, 195 leaves, 11 steps):

$$\frac{5^{-1-m} e^{5a} x^m \Gamma(1+m, -5bx)}{32b(-bx)^m} - \frac{3^{-1-m} e^{3a} x^m \Gamma(1+m, -3bx)}{32b(-bx)^m} - \frac{e^a x^m \Gamma(1+m, -bx)}{16b(-bx)^m} - \frac{x^m \Gamma(1+m, bx)}{16be^a (bx)^m} - \frac{3^{-1-m} x^m \Gamma(1+m, 3bx)}{32be^{3a} (bx)^m} + \frac{5^{-1-m} x^m \Gamma(1+m, 5bx)}{32be^{5a} (bx)^m}$$

Result(type 8, 22 leaves):

$$\int x^m \cosh(bx + a)^2 \sinh(bx + a)^3 dx$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int x^2 \cosh(bx + a)^3 \sinh(bx + a)^3 dx$$

Optimal(type 3, 93 leaves, 8 steps):

$$-\frac{3 \cosh(2bx + 2a)}{128b^3} - \frac{3x^2 \cosh(2bx + 2a)}{64b} + \frac{\cosh(6bx + 6a)}{3456b^3} + \frac{x^2 \cosh(6bx + 6a)}{192b} + \frac{3x \sinh(2bx + 2a)}{64b^2} - \frac{x \sinh(6bx + 6a)}{576b^2}$$

Result(type 3, 357 leaves):

$$\frac{1}{b^3} \left( \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)^4}{6} - \frac{(bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a)^2}{12} - \frac{(bx+a)^2 \cosh(bx+a)^2}{12} - \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^5}{18} + \frac{(bx+a) \sinh(bx+a) \cosh(bx+a)^3}{18} + \frac{\cosh(bx+a)^4 \sinh(bx+a)^2}{108} - \frac{\cosh(bx+a)^2 \sinh(bx+a)^2}{216} - \frac{5 \cosh(bx+a)^2}{108} + \frac{(bx+a) \cosh(bx+a) \sinh(bx+a)}{12} + \frac{(bx+a)^2}{24} - 2a \left( \frac{(bx+a) \sinh(bx+a)^2 \cosh(bx+a)^4}{6} - \frac{(bx+a) \sinh(bx+a)^2 \cosh(bx+a)^2}{12} - \frac{\cosh(bx+a)^2 (bx+a)}{12} - \frac{\sinh(bx+a) \cosh(bx+a)^5}{36} + \frac{\sinh(bx+a) \cosh(bx+a)^3}{36} + \frac{\cosh(bx+a) \sinh(bx+a)}{24} + \frac{bx}{24} + \frac{a}{24} \right) + a^2 \left( \frac{\cosh(bx+a)^4 \sinh(bx+a)^2}{6} - \frac{\cosh(bx+a)^2 \sinh(bx+a)^2}{12} - \frac{\cosh(bx+a)^2}{12} \right) \right)$$

Problem 92: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{sech}(bx + a)^2 \sinh(bx + a) dx$$

Optimal(type 4, 62 leaves, 6 steps):

$$\frac{4x \arctan(e^{bx+a})}{b^2} - \frac{2 \operatorname{Ipolylog}(2, -e^{bx+a})}{b^3} + \frac{2 \operatorname{Ipolylog}(2, e^{bx+a})}{b^3} - \frac{x^2 \operatorname{sech}(bx + a)}{b}$$

Result(type 4, 153 leaves):

$$-\frac{2x^2 e^{bx+a}}{b(1+e^{2bx+2a})} - \frac{2I \ln(1+Ie^{bx+a})x}{b^2} - \frac{2I \ln(1+Ie^{bx+a})a}{b^3} + \frac{2I \ln(1-Ie^{bx+a})x}{b^2} + \frac{2I \ln(1-Ie^{bx+a})a}{b^3} - \frac{2I \operatorname{dilog}(1+Ie^{bx+a})}{b^3} + \frac{2I \operatorname{dilog}(1-Ie^{bx+a})}{b^3} - \frac{4a \arctan(e^{bx+a})}{b^3}$$

Problem 109: Result more than twice size of optimal antiderivative.

$$\int x \cosh(bx+a)^2 \operatorname{csch}(bx+a) dx$$

Optimal (type 4, 63 leaves, 8 steps):

$$-\frac{2x \operatorname{arctanh}(e^{bx+a})}{b} + \frac{x \cosh(bx+a)}{b} - \frac{\operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{\operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{\sinh(bx+a)}{b^2}$$

Result (type 4, 138 leaves):

$$\frac{(bx-1)e^{bx+a}}{2b^2} + \frac{(bx+1)e^{-bx-a}}{2b^2} - \frac{\ln(e^{bx+a}+1)x}{b} - \frac{\ln(e^{bx+a}+1)a}{b^2} - \frac{\operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{\ln(-e^{bx+a}+1)x}{b} + \frac{\ln(-e^{bx+a}+1)a}{b^2} + \frac{\operatorname{polylog}(2, e^{bx+a})}{b^2} + \frac{2a \operatorname{arctanh}(e^{bx+a})}{b^2}$$

Problem 115: Result more than twice size of optimal antiderivative.

$$\int x^3 \cosh(bx+a)^2 \operatorname{csch}(bx+a)^2 dx$$

Optimal (type 4, 83 leaves, 7 steps):

$$-\frac{x^3}{b} + \frac{x^4}{4} - \frac{x^3 \operatorname{coth}(bx+a)}{b} + \frac{3x^2 \ln(1-e^{2bx+2a})}{b^2} + \frac{3x \operatorname{polylog}(2, e^{2bx+2a})}{b^3} - \frac{3 \operatorname{polylog}(3, e^{2bx+2a})}{2b^4}$$

Result (type 4, 197 leaves):

$$\frac{x^4}{4} - \frac{2x^3}{(e^{2bx+2a}-1)b} - \frac{6a^2 \ln(e^{bx+a})}{b^4} + \frac{3a^2 \ln(e^{bx+a}-1)}{b^4} - \frac{2x^3}{b} + \frac{6a^2 x}{b^3} + \frac{4a^3}{b^4} + \frac{3 \ln(e^{bx+a}+1)x^2}{b^2} + \frac{6x \operatorname{polylog}(2, -e^{bx+a})}{b^3} - \frac{6 \operatorname{polylog}(3, -e^{bx+a})}{b^4} + \frac{3 \ln(-e^{bx+a}+1)x^2}{b^2} - \frac{3 \ln(-e^{bx+a}+1)a^2}{b^4} + \frac{6x \operatorname{polylog}(2, e^{bx+a})}{b^3} - \frac{6 \operatorname{polylog}(3, e^{bx+a})}{b^4}$$

Problem 124: Result more than twice size of optimal antiderivative.

$$\int x^3 \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 dx$$

Optimal (type 4, 161 leaves, 13 steps):

$$-\frac{3x^2}{2b^2} + \frac{x^3}{2b} - \frac{x^4}{4} - \frac{3x^2 \operatorname{coth}(bx+a)}{2b^2} - \frac{x^3 \operatorname{coth}(bx+a)^2}{2b} + \frac{3x \ln(1-e^{2bx+2a})}{b^3} + \frac{x^3 \ln(1-e^{2bx+2a})}{b} + \frac{3 \operatorname{polylog}(2, e^{2bx+2a})}{2b^4}$$

$$+ \frac{3x^2 \operatorname{polylog}(2, e^{2bx+2a})}{2b^2} - \frac{3x \operatorname{polylog}(3, e^{2bx+2a})}{2b^3} + \frac{3 \operatorname{polylog}(4, e^{2bx+2a})}{4b^4}$$

Result(type 4, 374 leaves):

$$\begin{aligned} & -\frac{6ax}{b^3} + \frac{6a \ln(e^{bx+a})}{b^4} - \frac{3a \ln(e^{bx+a} - 1)}{b^4} - \frac{a^3 \ln(e^{bx+a} - 1)}{b^4} - \frac{3x^2}{b^2} + \frac{3 \operatorname{polylog}(2, -e^{bx+a})}{b^4} + \frac{3 \operatorname{polylog}(2, e^{bx+a})}{b^4} + \frac{6 \operatorname{polylog}(4, -e^{bx+a})}{b^4} \\ & + \frac{6 \operatorname{polylog}(4, e^{bx+a})}{b^4} - \frac{x^4}{4} + \frac{3x^2 \operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{3x^2 \operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{6x \operatorname{polylog}(3, -e^{bx+a})}{b^3} - \frac{6x \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{3a^2}{b^4} \\ & + \frac{3 \ln(e^{bx+a} + 1)x}{b^3} + \frac{3 \ln(-e^{bx+a} + 1)x}{b^3} + \frac{3 \ln(-e^{bx+a} + 1)a}{b^4} + \frac{\ln(e^{bx+a} + 1)x^3}{b} + \frac{\ln(-e^{bx+a} + 1)x^3}{b} + \frac{\ln(-e^{bx+a} + 1)a^3}{b^4} \\ & - \frac{x^2(2e^{2bx+2a}bx + 3e^{2bx+2a} - 3)}{b^2(e^{2bx+2a} - 1)^2} - \frac{3a^4}{2b^4} - \frac{2a^3x}{b^3} + \frac{2a^3 \ln(e^{bx+a})}{b^4} \end{aligned}$$

Problem 125: Result more than twice size of optimal antiderivative.

$$\int x \cosh(bx+a)^3 \operatorname{csch}(bx+a)^3 dx$$

Optimal(type 4, 72 leaves, 7 steps):

$$\frac{x}{2b} - \frac{x^2}{2} - \frac{\operatorname{coth}(bx+a)}{2b^2} - \frac{x \operatorname{coth}(bx+a)^2}{2b} + \frac{x \ln(1 - e^{2bx+2a})}{b} + \frac{\operatorname{polylog}(2, e^{2bx+2a})}{2b^2}$$

Result(type 4, 163 leaves):

$$\begin{aligned} & -\frac{x^2}{2} - \frac{2e^{2bx+2a}bx + e^{2bx+2a} - 1}{b^2(e^{2bx+2a} - 1)^2} - \frac{2ax}{b} - \frac{a^2}{b^2} + \frac{\ln(e^{bx+a} + 1)x}{b} + \frac{\operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{\ln(-e^{bx+a} + 1)x}{b} + \frac{\ln(-e^{bx+a} + 1)a}{b^2} \\ & + \frac{\operatorname{polylog}(2, e^{bx+a})}{b^2} + \frac{2a \ln(e^{bx+a})}{b^2} - \frac{a \ln(e^{bx+a} - 1)}{b^2} \end{aligned}$$

Problem 128: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a) dx$$

Optimal(type 4, 88 leaves, 8 steps):

$$-\frac{2x^2 \operatorname{arctanh}(e^{2bx+2a})}{b} - \frac{x \operatorname{polylog}(2, -e^{2bx+2a})}{b^2} + \frac{x \operatorname{polylog}(2, e^{2bx+2a})}{b^2} + \frac{\operatorname{polylog}(3, -e^{2bx+2a})}{2b^3} - \frac{\operatorname{polylog}(3, e^{2bx+2a})}{2b^3}$$

Result(type 4, 185 leaves):

$$\begin{aligned} & \frac{a^2 \ln(e^{bx+a} - 1)}{b^3} - \frac{\ln(-e^{bx+a} + 1)a^2}{b^3} - \frac{x^2 \ln(1 + e^{2bx+2a})}{b} - \frac{x \operatorname{polylog}(2, -e^{2bx+2a})}{b^2} + \frac{\ln(e^{bx+a} + 1)x^2}{b} + \frac{2x \operatorname{polylog}(2, -e^{bx+a})}{b^2} \\ & + \frac{\ln(-e^{bx+a} + 1)x^2}{b} + \frac{2x \operatorname{polylog}(2, e^{bx+a})}{b^2} + \frac{\operatorname{polylog}(3, -e^{2bx+2a})}{2b^3} - \frac{2 \operatorname{polylog}(3, -e^{bx+a})}{b^3} - \frac{2 \operatorname{polylog}(3, e^{bx+a})}{b^3} \end{aligned}$$

Problem 131: Unable to integrate problem.

$$\int x^3 \operatorname{csch}(bx+a) \operatorname{sech}(bx+a)^2 dx$$

Optimal(type 4, 206 leaves, 21 steps):

$$\begin{aligned} & -\frac{6x^2 \arctan(e^{bx+a})}{b^2} - \frac{2x^3 \operatorname{arctanh}(e^{bx+a})}{b} - \frac{3x^2 \operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{6Ix \operatorname{polylog}(2, -Ie^{bx+a})}{b^3} - \frac{6Ix \operatorname{polylog}(2, Ie^{bx+a})}{b^3} + \frac{3x^2 \operatorname{polylog}(2, e^{bx+a})}{b^2} \\ & + \frac{6x \operatorname{polylog}(3, -e^{bx+a})}{b^3} - \frac{6I \operatorname{polylog}(3, -Ie^{bx+a})}{b^4} + \frac{6I \operatorname{polylog}(3, Ie^{bx+a})}{b^4} - \frac{6x \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{6 \operatorname{polylog}(4, -e^{bx+a})}{b^4} \\ & + \frac{6 \operatorname{polylog}(4, e^{bx+a})}{b^4} + \frac{x^3 \operatorname{sech}(bx+a)}{b} \end{aligned}$$

Result(type 8, 95 leaves):

$$\frac{2x^3 e^{bx+a}}{b((e^{bx+a})^2 + 1)} + 8 \left( \int \frac{x^2 e^{bx+a} ((e^{bx+a})^2 bx + bx - 3(e^{bx+a})^2 + 3)}{4b((e^{bx+a})^2 + 1)((e^{bx+a})^2 - 1)} dx \right)$$

Problem 139: Unable to integrate problem.

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^2 dx$$

Optimal(type 4, 181 leaves, 29 steps):

$$\begin{aligned} & \frac{4x \arctan(e^{bx+a})}{b^2} + \frac{3x^2 \operatorname{arctanh}(e^{bx+a})}{b} - \frac{\operatorname{arctanh}(\cosh(bx+a))}{b^3} - \frac{x \operatorname{csch}(bx+a)}{b^2} + \frac{3x \operatorname{polylog}(2, -e^{bx+a})}{b^2} - \frac{2I \operatorname{polylog}(2, -Ie^{bx+a})}{b^3} \\ & + \frac{2I \operatorname{polylog}(2, Ie^{bx+a})}{b^3} - \frac{3x \operatorname{polylog}(2, e^{bx+a})}{b^2} - \frac{3 \operatorname{polylog}(3, -e^{bx+a})}{b^3} + \frac{3 \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{3x^2 \operatorname{sech}(bx+a)}{2b} \\ & - \frac{x^2 \operatorname{csch}(bx+a)^2 \operatorname{sech}(bx+a)}{2b} \end{aligned}$$

Result(type 8, 168 leaves):

$$\begin{aligned} & -\frac{x e^{bx+a} (3(e^{bx+a})^4 xb - 2(e^{bx+a})^2 bx + 2(e^{bx+a})^4 + 3bx - 2)}{b^2((e^{bx+a})^2 - 1)^2((e^{bx+a})^2 + 1)} + 32 \left( \int \right. \\ & \left. - \frac{e^{bx+a} (3b^2 x^2 (e^{bx+a})^2 + 3b^2 x^2 - 4(e^{bx+a})^2 bx + 4bx - 2(e^{bx+a})^2 - 2)}{32b^2((e^{bx+a})^2 - 1)((e^{bx+a})^2 + 1)} dx \right) \end{aligned}$$

Problem 140: Result more than twice size of optimal antiderivative.

$$\int x^2 \operatorname{csch}(bx+a)^3 \operatorname{sech}(bx+a)^3 dx$$

Optimal(type 4, 144 leaves, 10 steps):

$$\frac{4x^2 \operatorname{arctanh}(e^{2bx+2a})}{b} - \frac{\operatorname{arctanh}(\cosh(2bx+2a))}{b^3} - \frac{2x \operatorname{csch}(2bx+2a)}{b^2} - \frac{2x^2 \operatorname{coth}(2bx+2a) \operatorname{csch}(2bx+2a)}{b} + \frac{2x \operatorname{polylog}(2, -e^{2bx+2a})}{b^2}$$

$$-\frac{2x \operatorname{polylog}(2, e^{2bx+2a})}{b^2} - \frac{\operatorname{polylog}(3, -e^{2bx+2a})}{b^3} + \frac{\operatorname{polylog}(3, e^{2bx+2a})}{b^3}$$

Result(type 4, 298 leaves):

$$\begin{aligned} & -\frac{4xe^{2bx+2a}(e^{4bx+4a}xb + e^{4bx+4a} + bx - 1)}{b^2(e^{2bx+2a} - 1)^2(1 + e^{2bx+2a})^2} + \frac{2\ln(-e^{bx+a} + 1)a^2}{b^3} - \frac{2a^2\ln(e^{bx+a} - 1)}{b^3} - \frac{2\ln(-e^{bx+a} + 1)x^2}{b} - \frac{4x \operatorname{polylog}(2, e^{bx+a})}{b^2} \\ & + \frac{2x^2\ln(1 + e^{2bx+2a})}{b} + \frac{2x \operatorname{polylog}(2, -e^{2bx+2a})}{b^2} - \frac{2\ln(e^{bx+a} + 1)x^2}{b} - \frac{4x \operatorname{polylog}(2, -e^{bx+a})}{b^2} + \frac{4 \operatorname{polylog}(3, -e^{bx+a})}{b^3} \\ & + \frac{4 \operatorname{polylog}(3, e^{bx+a})}{b^3} - \frac{\operatorname{polylog}(3, -e^{2bx+2a})}{b^3} + \frac{\ln(e^{bx+a} + 1)}{b^3} + \frac{\ln(e^{bx+a} - 1)}{b^3} - \frac{\ln(1 + e^{2bx+2a})}{b^3} \end{aligned}$$

Problem 141: Unable to integrate problem.

$$\int x \cosh(bx + a)^{5/2} \sinh(bx + a) \, dx$$

Optimal(type 4, 99 leaves, 4 steps):

$$\begin{aligned} & \frac{2x \cosh(bx + a)^{7/2}}{7b} + \frac{20 \operatorname{I} \sqrt{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right)^2} \operatorname{EllipticF}\left(\operatorname{I} \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{147 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b^2} - \frac{4 \cosh(bx + a)^{5/2} \sinh(bx + a)}{49 b^2} \\ & - \frac{20 \sinh(bx + a) \sqrt{\cosh(bx + a)}}{147 b^2} \end{aligned}$$

Result(type 8, 18 leaves):

$$\int x \cosh(bx + a)^{5/2} \sinh(bx + a) \, dx$$

Problem 142: Unable to integrate problem.

$$\int x \sinh(bx + a) \sqrt{\cosh(bx + a)} \, dx$$

Optimal(type 4, 80 leaves, 3 steps):

$$\frac{2x \cosh(bx + a)^{3/2}}{3b} + \frac{4 \operatorname{I} \sqrt{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right)^2} \operatorname{EllipticF}\left(\operatorname{I} \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right)}{9 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b^2} - \frac{4 \sinh(bx + a) \sqrt{\cosh(bx + a)}}{9 b^2}$$

Result(type 8, 18 leaves):

$$\int x \sinh(bx + a) \sqrt{\cosh(bx + a)} \, dx$$

Problem 143: Unable to integrate problem.

$$\int x \operatorname{sech}(bx+a)^9 / 2 \sinh(bx+a) dx$$

Optimal(type 4, 115 leaves, 5 steps):

$$\begin{aligned} & -\frac{2x \operatorname{sech}(bx+a)^7 / 2}{7b} + \frac{4 \operatorname{sech}(bx+a)^5 / 2 \sinh(bx+a)}{35b^2} + \frac{12 \sinh(bx+a) \sqrt{\operatorname{sech}(bx+a)}}{35b^2} \\ & + \frac{12 \operatorname{I} \sqrt{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right)^2} \operatorname{EllipticE}\left(\operatorname{I} \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{\cosh(bx+a)} \sqrt{\operatorname{sech}(bx+a)}}{35 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b^2} \end{aligned}$$

Result(type 8, 18 leaves):

$$\int x \operatorname{sech}(bx+a)^9 / 2 \sinh(bx+a) dx$$

Problem 144: Result more than twice size of optimal antiderivative.

$$\int x \sqrt{\operatorname{sech}(bx+a)} \sinh(bx+a) dx$$

Optimal(type 4, 77 leaves, 3 steps):

$$\frac{2x}{b \sqrt{\operatorname{sech}(bx+a)}} + \frac{4 \operatorname{I} \sqrt{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right)^2} \operatorname{EllipticE}\left(\operatorname{I} \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{\cosh(bx+a)} \sqrt{\operatorname{sech}(bx+a)}}{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b^2}$$

Result(type 4, 249 leaves):

$$\begin{aligned} & \frac{(bx-2) \left( (e^{bx+a})^2 + 1 \right) \sqrt{2} \sqrt{\frac{e^{bx+a}}{(e^{bx+a})^2 + 1}}}{b^2 e^{bx+a}} - \frac{1}{b^2 e^{bx+a}} \left( 2 \left( -\frac{2 \left( (e^{bx+a})^2 + 1 \right)}{\sqrt{\left( (e^{bx+a})^2 + 1 \right) e^{bx+a}}} \right. \right. \\ & \left. \left. + \frac{\operatorname{I} \sqrt{-1(e^{bx+a}+1)} \sqrt{2} \sqrt{\operatorname{I}(e^{bx+a}-1)} \sqrt{\operatorname{I} e^{bx+a}} \left( -2 \operatorname{I} \operatorname{EllipticE}\left(\sqrt{-1(e^{bx+a}+1)}, \frac{\sqrt{2}}{2}\right) + \operatorname{I} \operatorname{EllipticF}\left(\sqrt{-1(e^{bx+a}+1)}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{(e^{bx+a})^3 + e^{bx+a}}} \right) \right) \\ & \left. \sqrt{2} \sqrt{\frac{e^{bx+a}}{(e^{bx+a})^2 + 1}} \sqrt{\left( (e^{bx+a})^2 + 1 \right) e^{bx+a}} \right) \end{aligned}$$

Problem 145: Unable to integrate problem.

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}} dx$$

Optimal(type 4, 96 leaves, 4 steps):

$$\frac{2x}{3b \operatorname{sech}(bx+a)^{3/2}} - \frac{4 \sinh(bx+a)}{9b^2 \sqrt{\operatorname{sech}(bx+a)}} + \frac{4 \operatorname{I} \sqrt{\cosh\left(\frac{a}{2} + \frac{bx}{2}\right)^2} \operatorname{EllipticF}\left(\operatorname{I} \sinh\left(\frac{a}{2} + \frac{bx}{2}\right), \sqrt{2}\right) \sqrt{\cosh(bx+a)} \sqrt{\operatorname{sech}(bx+a)}}{9 \cosh\left(\frac{a}{2} + \frac{bx}{2}\right) b^2}$$

Result(type 8, 18 leaves):

$$\int \frac{x \sinh(bx+a)}{\sqrt{\operatorname{sech}(bx+a)}} dx$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{x \cosh(bx+a)}{\sqrt{\sinh(bx+a)}} dx$$

Optimal(type 4, 92 leaves, 3 steps):

$$\frac{2x \sqrt{\sinh(bx+a)}}{b} - \frac{4 \operatorname{I} \sqrt{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right), \sqrt{2}\right) \sqrt{\sinh(bx+a)}}{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right) b^2 \sqrt{\operatorname{I} \sinh(bx+a)}}$$

Result(type 4, 228 leaves):

$$\frac{(bx-2) \left( (e^{bx+a})^2 - 1 \right) \sqrt{2}}{b^2 \sqrt{\frac{(e^{bx+a})^2 - 1}{e^{bx+a}}} e^{bx+a}} + \frac{1}{b^2 \sqrt{\frac{(e^{bx+a})^2 - 1}{e^{bx+a}}} e^{bx+a}} \left( 2 \left( \frac{2 \left( (e^{bx+a})^2 - 1 \right)}{\sqrt{\left( (e^{bx+a})^2 - 1 \right) e^{bx+a}}} \right. \right. \\ \left. \left. - \frac{\sqrt{e^{bx+a} + 1} \sqrt{-2e^{bx+a} + 2} \sqrt{-e^{bx+a}} \left( -2 \operatorname{EllipticE}\left(\sqrt{e^{bx+a} + 1}, \frac{\sqrt{2}}{2}\right) + \operatorname{EllipticF}\left(\sqrt{e^{bx+a} + 1}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{(e^{bx+a})^3 - e^{bx+a}}} \right) \sqrt{2} \sqrt{\left( (e^{bx+a})^2 - 1 \right) e^{bx+a}} \right)$$

Problem 147: Unable to integrate problem.

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^{7/2} dx$$

Optimal(type 4, 111 leaves, 4 steps):

$$-\frac{4 \cosh(bx+a) \operatorname{csch}(bx+a)^{3/2}}{15b^2} - \frac{2x \operatorname{csch}(bx+a)^{5/2}}{5b}$$

$$\frac{4 \operatorname{I} \sqrt{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(bx+a)} \sqrt{\operatorname{I} \sinh(bx+a)}}{15 \sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right) b^2}$$

Result(type 8, 18 leaves):

$$\int x \cosh(bx+a) \operatorname{csch}(bx+a)^7 / 2 \, dx$$

Problem 148: Result more than twice size of optimal antiderivative.

$$\int x \cosh(bx+a) \sqrt{\operatorname{csch}(bx+a)} \, dx$$

Optimal(type 4, 92 leaves, 3 steps):

$$\frac{2x}{b \sqrt{\operatorname{csch}(bx+a)}} - \frac{4 \operatorname{I} \sqrt{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right), \sqrt{2}\right)}{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right) b^2 \sqrt{\operatorname{csch}(bx+a)} \sqrt{\operatorname{I} \sinh(bx+a)}}$$

Result(type 4, 228 leaves):

$$\frac{(bx-2) \left( (e^{bx+a})^2 - 1 \right) \sqrt{2} \sqrt{\frac{e^{bx+a}}{(e^{bx+a})^2 - 1}}}{b^2 e^{bx+a}} + \frac{1}{b^2 e^{bx+a}} \left( 2 \left( \frac{2 \left( (e^{bx+a})^2 - 1 \right)}{\sqrt{\left( (e^{bx+a})^2 - 1 \right) e^{bx+a}}} \right) \right. \\ \left. - \frac{\sqrt{e^{bx+a} + 1} \sqrt{-2e^{bx+a} + 2} \sqrt{-e^{bx+a}} \left( -2 \operatorname{EllipticE}\left(\sqrt{e^{bx+a} + 1}, \frac{\sqrt{2}}{2}\right) + \operatorname{EllipticF}\left(\sqrt{e^{bx+a} + 1}, \frac{\sqrt{2}}{2}\right) \right)}{\sqrt{(e^{bx+a})^3 - e^{bx+a}}} \right) \\ \left. \sqrt{2} \sqrt{\frac{e^{bx+a}}{(e^{bx+a})^2 - 1}} \sqrt{\left( (e^{bx+a})^2 - 1 \right) e^{bx+a}} \right)$$

Problem 149: Unable to integrate problem.

$$\int \frac{x \cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}} \, dx$$

Optimal(type 4, 111 leaves, 4 steps):



$$\frac{2x}{3b \operatorname{csch}(bx+a)^{3/2}} - \frac{4 \cosh(bx+a)}{9b^2 \sqrt{\operatorname{csch}(bx+a)}} + \frac{4 \operatorname{I} \sqrt{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right)^2} \operatorname{EllipticF}\left(\cos\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right), \sqrt{2}\right) \sqrt{\operatorname{csch}(bx+a)} \sqrt{\operatorname{I} \sinh(bx+a)}}{9 \sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right) b^2}$$

Result(type 8, 18 leaves):

$$\int \frac{x \cosh(bx+a)}{\sqrt{\operatorname{csch}(bx+a)}} dx$$

Problem 150: Unable to integrate problem.

$$\int \frac{x \cosh(bx+a)}{\operatorname{csch}(bx+a)^{3/2}} dx$$

Optimal(type 4, 111 leaves, 4 steps):

$$\frac{2x}{5b \operatorname{csch}(bx+a)^{5/2}} - \frac{4 \cosh(bx+a)}{25b^2 \operatorname{csch}(bx+a)^{3/2}} + \frac{12 \operatorname{I} \sqrt{\sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right)^2} \operatorname{EllipticE}\left(\cos\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right), \sqrt{2}\right)}{25 \sin\left(\frac{1a}{2} + \frac{\pi}{4} + \frac{1bx}{2}\right) b^2 \sqrt{\operatorname{csch}(bx+a)} \sqrt{\operatorname{I} \sinh(bx+a)}}$$

Result(type 8, 18 leaves):

$$\int \frac{x \cosh(bx+a)}{\operatorname{csch}(bx+a)^{3/2}} dx$$

Problem 151: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\sinh(x) \tanh(x)} dx$$

Optimal(type 3, 11 leaves, 3 steps):

$$2 \coth(x) \sqrt{\sinh(x) \tanh(x)}$$

Result(type 3, 41 leaves):

$$\frac{\sqrt{2} \sqrt{\frac{(e^{2x}-1)^2 e^{-x}}{e^{2x}+1}} (e^{2x}+1)}{e^{2x}-1}$$

Problem 152: Unable to integrate problem.

$$\int (\sinh(x) \tanh(x))^{3/2} dx$$

Optimal(type 3, 23 leaves, 4 steps):

$$\frac{8 \operatorname{csch}(x) \sqrt{\sinh(x) \tanh(x)}}{3} + \frac{2 \sinh(x) \sqrt{\sinh(x) \tanh(x)}}{3}$$

Result(type 8, 9 leaves):

$$\int (\sinh(x) \tanh(x))^3 / 2 \, dx$$

Problem 153: Result more than twice size of optimal antiderivative.

$$\int \frac{b + c + \cosh(x)}{a - b \sinh(x)} \, dx$$

Optimal(type 3, 47 leaves, 7 steps):

$$-\frac{\ln(a - b \sinh(x))}{b} + \frac{2(b + c) \operatorname{arctanh}\left(\frac{b + a \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}}$$

Result(type 3, 118 leaves):

$$\frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{b} - \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 + 2b \tanh\left(\frac{x}{2}\right) - a\right)}{b} + \frac{2b \operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 + b^2}}\right)}{\sqrt{a^2 + b^2}} + \frac{2 \operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right) + 2b}{2\sqrt{a^2 + b^2}}\right)c}{\sqrt{a^2 + b^2}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b}$$

Problem 156: Maple result simpler than optimal antiderivative, IF it can be verified!

$$\int \sqrt{a \cosh(x) + b \sinh(x)} \, dx$$

Optimal(type 4, 89 leaves, 2 steps):

$$\frac{-2I \sqrt{\cos\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right)^2} \operatorname{EllipticE}\left(\sin\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right), \sqrt{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}{\cos\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{a^2 - b^2}}}$$

Result(type 3, 32 leaves):

$$-\frac{\sqrt{a^2 - b^2} \cosh(x)}{\sqrt{-\sinh(x) \sqrt{a^2 - b^2}}}$$

Problem 157: Maple result simpler than optimal antiderivative, IF it can be verified!

$$\int \frac{1}{\sqrt{a \cosh(x) + b \sinh(x)}} dx$$

Optimal(type 4, 89 leaves, 2 steps):

$$\frac{-2I \sqrt{\cos\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right)^2} \operatorname{EllipticF}\left(\sin\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right), \sqrt{2}\right) \sqrt{\frac{a \cosh(x) + b \sinh(x)}{\sqrt{a^2 - b^2}}}}{\cos\left(\frac{Ix}{2} - \frac{\arctan(a, -Ib)}{2}\right) \sqrt{a \cosh(x) + b \sinh(x)}}$$

Result(type 3, 96 leaves):

$$\frac{\sqrt{-\sinh(x)^3 \sqrt{a^2 - b^2}} \arctan\left(\frac{\sqrt{\sinh(x) \sqrt{a^2 - b^2}} \cosh(x)}{\sqrt{-\sinh(x)^3 \sqrt{a^2 - b^2}}}\right)}{\sqrt{\sinh(x) \sqrt{a^2 - b^2}} \sinh(x) \sqrt{-\sinh(x) \sqrt{a^2 - b^2}}}$$

Problem 167: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{sech}(x) + b \tanh(x))^3} dx$$

Optimal(type 3, 50 leaves, 4 steps):

$$\frac{\ln(a + b \sinh(x))}{b^3} + \frac{-a^2 - b^2}{2b^3 (a + b \sinh(x))^2} + \frac{2a}{b^3 (a + b \sinh(x))}$$

Result(type 3, 240 leaves):

$$\begin{aligned} & -\frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{b^3} + \frac{2a \tanh\left(\frac{x}{2}\right)^3}{b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2} - \frac{2 \tanh\left(\frac{x}{2}\right)^3}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2 a} - \frac{6 \tanh\left(\frac{x}{2}\right)^2}{b \left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2} \\ & + \frac{2b \tanh\left(\frac{x}{2}\right)^2}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2 a^2} - \frac{2a \tanh\left(\frac{x}{2}\right)}{b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2} + \frac{2 \tanh\left(\frac{x}{2}\right)}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^2 a} \\ & + \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)}{b^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b^3} \end{aligned}$$

Problem 168: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{sech}(x) + b \tanh(x))^4} dx$$

Optimal (type 3, 135 leaves, 8 steps):

$$\frac{x}{b^4} + \frac{a(2a^2 + 3b^2) \operatorname{arctanh}\left(\frac{b - a \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 + b^2}}\right)}{b^4 (a^2 + b^2)^{3/2}} - \frac{\cosh(x)^3}{3b(a + b \sinh(x))^3} + \frac{a \cosh(x)^3}{2b(a^2 + b^2)(a + b \sinh(x))^2} - \frac{\cosh(x)(2a^2 + 2b^2 + ab \sinh(x))}{2b^3(a^2 + b^2)(a + b \sinh(x))}$$

Result (type 3, 971 leaves):

$$\begin{aligned} & \frac{2b}{3\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} + \frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{b^4} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b^4} + \frac{2a \tanh\left(\frac{x}{2}\right)^5}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\ & - \frac{4b \tanh\left(\frac{x}{2}\right)^4}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} - \frac{2a \tanh\left(\frac{x}{2}\right)^3}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\ & + \frac{14b \tanh\left(\frac{x}{2}\right)^2}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} + \frac{8a \tanh\left(\frac{x}{2}\right)}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\ & + \frac{2a^4}{b^3\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} + \frac{5a^2}{3b\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} - \frac{2a^3 \operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2 + b^2}}\right)}{b^4 (a^2 + b^2)^{3/2}} \\ & - \frac{3a \operatorname{arctanh}\left(\frac{2a \tanh\left(\frac{x}{2}\right) - 2b}{2\sqrt{a^2 + b^2}}\right)}{b^2 (a^2 + b^2)^{3/2}} + \frac{a^3 \tanh\left(\frac{x}{2}\right)^5}{b^2\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} + \frac{2b^2 \tanh\left(\frac{x}{2}\right)^5}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 a (a^2 + b^2)} \\ & + \frac{2a^4 \tanh\left(\frac{x}{2}\right)^4}{b^3\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} - \frac{3a^2 \tanh\left(\frac{x}{2}\right)^4}{b\left(a \tanh\left(\frac{x}{2}\right)^2 - 2b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \end{aligned}$$

$$\begin{aligned}
& - \frac{4 b^3 \tanh\left(\frac{x}{2}\right)^4}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2) a^2} - \frac{12 a^3 \tanh\left(\frac{x}{2}\right)^3}{b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\
& + \frac{8 b^2 \tanh\left(\frac{x}{2}\right)^3}{3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 a (a^2 + b^2)} + \frac{8 b^4 \tanh\left(\frac{x}{2}\right)^3}{3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 a^3 (a^2 + b^2)} \\
& - \frac{4 a^4 \tanh\left(\frac{x}{2}\right)^2}{b^3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} + \frac{16 a^2 \tanh\left(\frac{x}{2}\right)^2}{b \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\
& + \frac{4 b^3 \tanh\left(\frac{x}{2}\right)^2}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 a^2 (a^2 + b^2)} + \frac{11 a^3 \tanh\left(\frac{x}{2}\right)}{b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 (a^2 + b^2)} \\
& + \frac{2 b^2 \tanh\left(\frac{x}{2}\right)}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^3 a (a^2 + b^2)}
\end{aligned}$$

Problem 169: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{sech}(x) + b \tanh(x))^5} dx$$

Optimal (type 3, 92 leaves, 4 steps):

$$\frac{\ln(a + b \sinh(x))}{b^5} - \frac{(a^2 + b^2)^2}{4 b^5 (a + b \sinh(x))^4} + \frac{4 a (a^2 + b^2)}{3 b^5 (a + b \sinh(x))^3} + \frac{-3 a^2 - b^2}{b^5 (a + b \sinh(x))^2} + \frac{4 a}{b^5 (a + b \sinh(x))}$$

Result (type 3, 720 leaves):

$$\begin{aligned}
& - \frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{b^5} + \frac{2 a^3 \tanh\left(\frac{x}{2}\right)^7}{b^4 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} - \frac{2 \tanh\left(\frac{x}{2}\right)^7}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a} - \frac{14 a^2 \tanh\left(\frac{x}{2}\right)^6}{b^3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} \\
& + \frac{6 b \tanh\left(\frac{x}{2}\right)^6}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^2} - \frac{6 a^3 \tanh\left(\frac{x}{2}\right)^5}{b^4 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} + \frac{104 a \tanh\left(\frac{x}{2}\right)^5}{3 b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4}
\end{aligned}$$

$$\begin{aligned}
& + \frac{2 \tanh\left(\frac{x}{2}\right)^5}{3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a} - \frac{8 b^2 \tanh\left(\frac{x}{2}\right)^5}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^3} + \frac{28 a^2 \tanh\left(\frac{x}{2}\right)^4}{b^3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} \\
& - \frac{100 \tanh\left(\frac{x}{2}\right)^4}{3 b \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} - \frac{28 b \tanh\left(\frac{x}{2}\right)^4}{3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^2} + \frac{4 b^3 \tanh\left(\frac{x}{2}\right)^4}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^4} \\
& + \frac{6 a^3 \tanh\left(\frac{x}{2}\right)^3}{b^4 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} - \frac{104 a \tanh\left(\frac{x}{2}\right)^3}{3 b^2 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} - \frac{2 \tanh\left(\frac{x}{2}\right)^3}{3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a} \\
& + \frac{8 b^2 \tanh\left(\frac{x}{2}\right)^3}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^3} - \frac{14 a^2 \tanh\left(\frac{x}{2}\right)^2}{b^3 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} + \frac{6 b \tanh\left(\frac{x}{2}\right)^2}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4 a^2} \\
& - \frac{2 a^3 \tanh\left(\frac{x}{2}\right)}{b^4 \left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)^4} + \frac{2 \tanh\left(\frac{x}{2}\right)}{\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right) a} + \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - 2 b \tanh\left(\frac{x}{2}\right) - a\right)}{b^5} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{b^5}
\end{aligned}$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int (\operatorname{sech}(x) + I \tanh(x))^5 dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$I \ln(I + \sinh(x)) - \frac{2I}{(1 - I \sinh(x))^2} + \frac{4I}{1 - I \sinh(x)}$$

Result (type 3, 81 leaves):

$$\begin{aligned}
& \frac{8 \left( \frac{\operatorname{sech}(x)^3}{4} + \frac{3 \operatorname{sech}(x)}{8} \right) \tanh(x)}{3} + 2 \arctan(e^x) + \frac{15 I \sinh(x)^2}{4 \cosh(x)^4} - \frac{5 I \sinh(x)^2}{4 \cosh(x)^2} - \frac{5 \sinh(x)}{3 \cosh(x)^4} - \frac{5 \sinh(x)^3}{\cosh(x)^4} + I \ln(\cosh(x)) - \frac{I \tanh(x)^2}{2} \\
& - \frac{I \tanh(x)^4}{4}
\end{aligned}$$

Problem 172: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\operatorname{sech}(x) - \operatorname{I} \tanh(x)} dx$$

Optimal(type 3, 9 leaves, 3 steps):

$$\operatorname{I} \ln(\operatorname{I} + \sinh(x))$$

Result(type 3, 32 leaves):

$$2 \operatorname{I} \ln\left(\tanh\left(\frac{x}{2}\right) + \operatorname{I}\right) - \operatorname{I} \ln\left(1 + \tanh\left(\frac{x}{2}\right)\right) - \operatorname{I} \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$$

Problem 174: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a \operatorname{coth}(x) + b \operatorname{csch}(x))^3} dx$$

Optimal(type 3, 48 leaves, 4 steps):

$$\frac{a^2 - b^2}{2a^3(b + a \cosh(x))^2} + \frac{2b}{a^3(b + a \cosh(x))} + \frac{\ln(b + a \cosh(x))}{a^3}$$

Result(type 3, 143 leaves):

$$\begin{aligned} & -\frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{a^3} + \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b + a + b\right)}{a^3} + \frac{2}{(a-b)\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b + a + b\right)^2} \\ & + \frac{2b}{a(a-b)\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b + a + b\right)^2} - \frac{2}{a^2\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b + a + b\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{a^3} \end{aligned}$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int (\operatorname{coth}(x) + \operatorname{csch}(x))^3 dx$$

Optimal(type 3, 18 leaves, 4 steps):

$$\frac{2}{1 - \cosh(x)} + \ln(1 - \cosh(x))$$

Result(type 3, 38 leaves):

$$\ln(\sinh(x)) - \frac{\operatorname{coth}(x)^2}{2} - \frac{3 \cosh(x)}{\sinh(x)^2} + \operatorname{coth}(x) \operatorname{csch}(x) - 2 \operatorname{arctanh}(e^x) - \frac{3 \cosh(x)^2}{2 \sinh(x)^2}$$

Problem 179: Result more than twice size of optimal antiderivative.

$$\int (-\coth(x) + \operatorname{csch}(x))^5 dx$$

Optimal(type 3, 24 leaves, 4 steps):

$$\frac{2}{(1 + \cosh(x))^2} - \frac{4}{1 + \cosh(x)} - \ln(1 + \cosh(x))$$

Result(type 3, 76 leaves):

$$-\ln(\sinh(x)) + \frac{\coth(x)^2}{2} + \frac{\coth(x)^4}{4} - \frac{5 \cosh(x)^3}{\sinh(x)^4} + \frac{5 \cosh(x)}{3 \sinh(x)^4} + \frac{8 \left( -\frac{\operatorname{csch}(x)^3}{4} + \frac{3 \operatorname{csch}(x)}{8} \right) \coth(x)}{3} - 2 \operatorname{arctanh}(e^x) + \frac{15 \cosh(x)^2}{4 \sinh(x)^4} + \frac{5 \cosh(x)^2}{4 \sinh(x)^2}$$

Problem 191: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^2}{(a \cosh(x) + b \sinh(x))^2} dx$$

Optimal(type 3, 67 leaves, 4 steps):

$$\frac{(a^2 + b^2)x}{(a^2 - b^2)^2} - \frac{2ab \ln(a \cosh(x) + b \sinh(x))}{(a^2 - b^2)^2} + \frac{b}{(a^2 - b^2)(a + b \tanh(x))}$$

Result(type 3, 148 leaves):

$$\begin{aligned} & -\frac{2b^2 a \tanh\left(\frac{x}{2}\right)}{(a-b)^2 (a+b)^2 \left(a + 2b \tanh\left(\frac{x}{2}\right) + a \tanh\left(\frac{x}{2}\right)^2\right)} + \frac{2b^4 \tanh\left(\frac{x}{2}\right)}{(a-b)^2 (a+b)^2 a \left(a + 2b \tanh\left(\frac{x}{2}\right) + a \tanh\left(\frac{x}{2}\right)^2\right)} \\ & -\frac{2ba \ln\left(a + 2b \tanh\left(\frac{x}{2}\right) + a \tanh\left(\frac{x}{2}\right)^2\right)}{(a-b)^2 (a+b)^2} + \frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{(a-b)^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{(a+b)^2} \end{aligned}$$

Problem 193: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^3}{(a \cosh(x) + b \sinh(x))^3} dx$$

Optimal(type 3, 102 leaves, 5 steps):

$$-\frac{b(3a^2 + b^2)x}{(a^2 - b^2)^3} - \frac{a}{2(a^2 - b^2)(b + a \coth(x))^2} + \frac{2ab}{(a^2 - b^2)^2(b + a \coth(x))} + \frac{a(a^2 + 3b^2) \ln(a \cosh(x) + b \sinh(x))}{(a^2 - b^2)^3}$$

Result(type 3, 403 leaves):

$$\frac{4a^4 b \tanh\left(\frac{x}{2}\right)^3}{(a-b)^3 (a+b)^3 \left(a + 2b \tanh\left(\frac{x}{2}\right) + a \tanh\left(\frac{x}{2}\right)^2\right)^2} - \frac{4a^2 b^3 \tanh\left(\frac{x}{2}\right)^3}{(a-b)^3 (a+b)^3 \left(a + 2b \tanh\left(\frac{x}{2}\right) + a \tanh\left(\frac{x}{2}\right)^2\right)^2}$$



$$\begin{aligned}
& - \frac{2 a^5 \tanh\left(\frac{x}{2}\right)^2}{(a-b)^3 (a+b)^3 \left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)^2} + \frac{12 a^3 b^2 \tanh\left(\frac{x}{2}\right)^2}{(a-b)^3 (a+b)^3 \left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)^2} \\
& - \frac{10 a \tanh\left(\frac{x}{2}\right)^2 b^4}{(a-b)^3 (a+b)^3 \left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)^2} + \frac{4 a^4 b \tanh\left(\frac{x}{2}\right)}{(a-b)^3 (a+b)^3 \left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)^2} \\
& - \frac{4 a^2 \tanh\left(\frac{x}{2}\right) b^3}{(a-b)^3 (a+b)^3 \left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)^2} + \frac{a^3 \ln\left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right)}{(a-b)^3 (a+b)^3} \\
& + \frac{3 a \ln\left(a+2 b \tanh\left(\frac{x}{2}\right)+a \tanh\left(\frac{x}{2}\right)^2\right) b^2}{(a-b)^3 (a+b)^3} - \frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{(a-b)^3} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{(a+b)^3}
\end{aligned}$$

Problem 199: Result more than twice size of optimal antiderivative.

$$\int \frac{A + C \sinh(x)}{b \cosh(x) + c \sinh(x)} dx$$

Optimal (type 3, 76 leaves, 3 steps):

$$-\frac{c C x}{b^2 - c^2} + \frac{b C \ln(b \cosh(x) + c \sinh(x))}{b^2 - c^2} + \frac{A \arctan\left(\frac{c \cosh(x) + b \sinh(x)}{\sqrt{b^2 - c^2}}\right)}{\sqrt{b^2 - c^2}}$$

Result (type 3, 180 leaves):

$$\begin{aligned}
& \frac{b C \ln\left(\tanh\left(\frac{x}{2}\right)^2 b + 2 c \tanh\left(\frac{x}{2}\right) + b\right)}{(b-c)(b+c)} + \frac{2 \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right) + 2 c}{2 \sqrt{b^2 - c^2}}\right) A b^2}{(b-c)(b+c) \sqrt{b^2 - c^2}} - \frac{2 \arctan\left(\frac{2 b \tanh\left(\frac{x}{2}\right) + 2 c}{2 \sqrt{b^2 - c^2}}\right) A c^2}{(b-c)(b+c) \sqrt{b^2 - c^2}} - \frac{2 C \ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{2 b - 2 c} \\
& - \frac{2 C \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2 b + 2 c}
\end{aligned}$$

Problem 205: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + b \cosh(x) + c \sinh(x))^2} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{2a \operatorname{arctanh}\left(\frac{c - (a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 + c^2}}\right)}{(a^2 - b^2 + c^2)^{3/2}} + \frac{-c \cosh(x) - b \sinh(x)}{(a^2 - b^2 + c^2)(a + b \cosh(x) + c \sinh(x))}$$

Result(type 3, 190 leaves):

$$-\frac{2\left(-\frac{(ab - b^2 + c^2) \tanh\left(\frac{x}{2}\right)}{a^3 - a^2b - ab^2 + ac^2 + b^3 - c^2b} - \frac{ac}{a^3 - a^2b - ab^2 + ac^2 + b^3 - c^2b}\right)}{a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - 2c \tanh\left(\frac{x}{2}\right) - a - b} - \frac{2a \operatorname{arctan}\left(\frac{2(a-b) \tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right)}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}}$$

Problem 208: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^2} dx$$

Optimal(type 3, 91 leaves, 2 steps):

$$\frac{c \cosh(x) + b \sinh(x)}{3\sqrt{b^2 - c^2}(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^2} + \frac{-c - \sinh(x)\sqrt{b^2 - c^2}}{3c(c \cosh(x) + b \sinh(x))\sqrt{b^2 - c^2}}$$

Result(type 3, 216 leaves):

$$\frac{2(\sqrt{b^2 - c^2} + b)\left(\frac{(\sqrt{b^2 - c^2} + b) \tanh\left(\frac{x}{2}\right)^2}{c^2} + \frac{(2b^2 - c^2 + 2\sqrt{b^2 - c^2}b) \tanh\left(\frac{x}{2}\right)}{c^3} + \frac{2(2\sqrt{b^2 - c^2}b^2 - \sqrt{b^2 - c^2}c^2 + 2b^3 - 2c^2b)}{3c^4}\right)}{c^2\left(\tanh\left(\frac{x}{2}\right)^2 + \frac{2\sqrt{(b-c)(b+c)} \tanh\left(\frac{x}{2}\right)}{c} + \frac{2b \tanh\left(\frac{x}{2}\right)}{c} + \frac{2\sqrt{(b-c)(b+c)}b}{c^2} + \frac{2b^2}{c^2} - 1\right)\left(\tanh\left(\frac{x}{2}\right) + \frac{\sqrt{(b-c)(b+c)}}{c} + \frac{b}{c}\right)}$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^4} dx$$

Optimal(type 3, 176 leaves, 4 steps):

$$\frac{c \cosh(x) + b \sinh(x)}{7\sqrt{b^2 - c^2}(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^4} + \frac{3(c \cosh(x) + b \sinh(x))}{35(b^2 - c^2)(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^3} + \frac{2(c \cosh(x) + b \sinh(x))}{35(b^2 - c^2)^{3/2}(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^2} - \frac{2(c + \sinh(x)\sqrt{b^2 - c^2})}{35c(b^2 - c^2)^{3/2}(c \cosh(x) + b \sinh(x))}$$

Result(type 3, 827 leaves):

$$\begin{aligned}
& \left( 2 \left( \frac{(8\sqrt{b^2-c^2}b^3 - 4\sqrt{b^2-c^2}bc^2 + 8b^4 - 8b^2c^2 + c^4) \tanh\left(\frac{x}{2}\right)^6}{c^2} \right. \right. \\
& + \frac{3(16\sqrt{b^2-c^2}b^4 - 12\sqrt{b^2-c^2}b^2c^2 + \sqrt{b^2-c^2}c^4 + 16b^5 - 20b^3c^2 + 5c^4b) \tanh\left(\frac{x}{2}\right)^5}{c^3} \\
& + \frac{2(80\sqrt{b^2-c^2}b^5 - 84\sqrt{b^2-c^2}b^3c^2 + 17\sqrt{b^2-c^2}bc^4 + 80b^6 - 124b^4c^2 + 49b^2c^4 - 3c^6) \tanh\left(\frac{x}{2}\right)^4}{c^4} \\
& + \frac{2(160b^7 - 288b^5c^2 + 150b^3c^4 - 20bc^6 + 160\sqrt{b^2-c^2}b^6 - 208\sqrt{b^2-c^2}b^4c^2 + 66\sqrt{b^2-c^2}b^2c^4 - 3\sqrt{b^2-c^2}c^6) \tanh\left(\frac{x}{2}\right)^3}{c^5} \\
& + \frac{1}{5c^6} \left( 3(640b^7\sqrt{b^2-c^2} - 992\sqrt{b^2-c^2}b^5c^2 + 440\sqrt{b^2-c^2}b^3c^4 - 50\sqrt{b^2-c^2}bc^6 + 640b^8 - 1312b^6c^2 + 856b^4c^4 - 186b^2c^6 \right. \\
& + 7c^8) \tanh\left(\frac{x}{2}\right)^2 \Big) + \frac{1}{5c^7} \left( (1280b^9 - 2944b^7c^2 + 2288b^5c^4 - 676b^3c^6 + 57bc^8 + 1280\sqrt{b^2-c^2}b^8 - 2304\sqrt{b^2-c^2}b^6c^2 + 1296\sqrt{b^2-c^2}b^4c^4 \right. \\
& - 236\sqrt{b^2-c^2}b^2c^6 + 7\sqrt{b^2-c^2}c^8) \tanh\left(\frac{x}{2}\right) \Big) + \frac{1}{35c^8} \left( 4(640\sqrt{b^2-c^2}b^9 - 1312\sqrt{b^2-c^2}b^7c^2 + 896\sqrt{b^2-c^2}b^5c^4 - 238\sqrt{b^2-c^2}b^3c^6 \right. \\
& + 21\sqrt{b^2-c^2}bc^8 + 640b^{10} - 1632b^8c^2 + 1472b^6c^4 - 562b^4c^6 + 85b^2c^8 - 3c^{10}) \Big) \Big) \Big/ \left( c^6 \left( \tanh\left(\frac{x}{2}\right)^2 + \frac{2\sqrt{b^2-c^2} \tanh\left(\frac{x}{2}\right)}{c} \right) \right. \\
& \left. + \frac{2b \tanh\left(\frac{x}{2}\right)}{c} + \frac{2\sqrt{b^2-c^2}b}{c^2} + \frac{2b^2}{c^2} - 1 \right)^3 \left( \tanh\left(\frac{x}{2}\right) + \frac{\sqrt{b^2-c^2}}{c} + \frac{b}{c} \right) \Big)
\end{aligned}$$

Problem 210: Maple result simpler than optimal antiderivative, IF it can be verified!

$$\int \sqrt{a + b \cosh(x) + c \sinh(x)} \, dx$$

Optimal(type 4, 125 leaves, 2 steps):

$$\frac{-2I \sqrt{\cos\left(\frac{Ix}{2} - \frac{\arctan(b, -Ic)}{2}\right)^2} \operatorname{EllipticE}\left(\sin\left(\frac{Ix}{2} - \frac{\arctan(b, -Ic)}{2}\right), \sqrt{2} \sqrt{\frac{\sqrt{b^2-c^2}}{a + \sqrt{b^2-c^2}}}\right) \sqrt{a + b \cosh(x) + c \sinh(x)}}{\cos\left(\frac{Ix}{2} - \frac{\arctan(b, -Ic)}{2}\right) \sqrt{\frac{a + b \cosh(x) + c \sinh(x)}{a + \sqrt{b^2-c^2}}}}$$

Result(type 3, 313 leaves):

$$\begin{aligned}
& \frac{(-b^2 + c^2) \cosh(x)}{\sqrt{b^2 - c^2} \sqrt{\frac{-\sinh(x) b^2 + \sinh(x) c^2 + a \sqrt{b^2 - c^2}}{\sqrt{b^2 - c^2}}}} \\
& + \frac{1}{(-\sinh(x) b^2 + \sinh(x) c^2 + a \sqrt{b^2 - c^2}) \sinh(x)} \left( \sqrt{\frac{(-\sinh(x) b^2 + \sinh(x) c^2 + a \sqrt{b^2 - c^2}) \sinh(x)^2}{\sqrt{b^2 - c^2}}} a \ln \left( \frac{1}{\sqrt{b^2 - c^2}} \right) \right. \\
& \left. \left( \sqrt{b^2 - c^2} \sqrt{\frac{-\sinh(x) b^2 + \sinh(x) c^2 + a \sqrt{b^2 - c^2}}{\sqrt{b^2 - c^2}}} \right) \left( \cosh(x) \sinh(x) (-b^2 + c^2) + \cosh(x) \sqrt{b^2 - c^2} a \right. \right. \\
& \left. \left. + \sqrt{\frac{(-b^2 + c^2) \sinh(x)^3}{\sqrt{b^2 - c^2}} + a \sinh(x)^2 \sqrt{b^2 - c^2}} \sqrt{\frac{(-b^2 + c^2) \sinh(x)}{\sqrt{b^2 - c^2}} + a} \right) \sqrt{b^2 - c^2} \right)
\end{aligned}$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int (b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^{5/2} dx$$

Optimal (type 3, 120 leaves, 3 steps):

$$\begin{aligned}
& \frac{2(c \cosh(x) + b \sinh(x)) (b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^{3/2}}{5} + \frac{64(b^2 - c^2)(c \cosh(x) + b \sinh(x))}{15 \sqrt{b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2}}} \\
& + \frac{16(c \cosh(x) + b \sinh(x)) \sqrt{b^2 - c^2} \sqrt{b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2}}}{15}
\end{aligned}$$

Result (type 3, 517 leaves):

$$\begin{aligned}
& \frac{-(b^2 - c^2)^{3/2} \cosh(x)^3}{3} - \frac{(-2b^2 + 2c^2)(-b^2 + c^2) \cosh(x)}{\sqrt{b^2 - c^2}} \\
& \frac{\sqrt{\frac{-\sinh(x) b^2 - \sinh(x) c^2 - b^2 + c^2}{\sqrt{b^2 - c^2}}}}{\sqrt{b^2 - c^2}} \\
& - \left( \left( \cosh(x) \sqrt{\sinh(x) \sqrt{b^2 - c^2} - \sqrt{b^2 - c^2}} \sqrt{-\sqrt{b^2 - c^2} \sinh(x)^3 + \sqrt{b^2 - c^2} \sinh(x)^2} (b^2 - c^2) - \sinh(x) (b^2 - c^2)^{3/2} \arctan \left( \frac{\sqrt{\sinh(x) \sqrt{b^2 - c^2} - \sqrt{b^2 - c^2}}}{\sqrt{-\sqrt{b^2 - c^2} \sinh(x)^3 + \sqrt{b^2 - c^2} \sinh(x)^2}} \right) \right. \right. \\
& \left. \left. + \sqrt{b^2 - c^2} \arctan \left( \frac{\sqrt{\sinh(x) \sqrt{b^2 - c^2} - \sqrt{b^2 - c^2}} \cosh(x)}{\sqrt{-\sqrt{b^2 - c^2} \sinh(x)^3 + \sqrt{b^2 - c^2} \sinh(x)^2}} \right) b^2 - \sqrt{b^2 - c^2} \arctan \left( \frac{\sqrt{\sinh(x) \sqrt{b^2 - c^2} - \sqrt{b^2 - c^2}} \cosh(x)}{\sqrt{-\sqrt{b^2 - c^2} \sinh(x)^3 + \sqrt{b^2 - c^2} \sinh(x)^2}} \right) c^2 \right)
\end{aligned}$$

$$\left. \sqrt{-\sqrt{b^2 - c^2} \sinh(x)^2 (-1 + \sinh(x))} \right) / \left( 2\sqrt{\sqrt{b^2 - c^2} (-1 + \sinh(x))} (-1 + \sinh(x)) \sinh(x) \sqrt{-\frac{\sinh(x) b^2 - \sinh(x) c^2 - b^2 + c^2}{\sqrt{b^2 - c^2}}} \right)$$

Problem 212: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^{3/2}} dx$$

Optimal(type 3, 128 leaves, 4 steps):

$$\frac{\arctan\left(\frac{(b^2 - c^2)^{1/4} \sinh(x + \text{Iarctan}(b, -Ic)) \sqrt{2}}{2\sqrt{\sqrt{b^2 - c^2} + \cosh(x + \text{Iarctan}(b, -Ic)) \sqrt{b^2 - c^2}}}\right) \sqrt{2}}{4(b^2 - c^2)^{3/4}} + \frac{c \cosh(x) + b \sinh(x)}{2\sqrt{b^2 - c^2} (b \cosh(x) + c \sinh(x) + \sqrt{b^2 - c^2})^{3/2}}$$

Result(type 1, 1 leaves):???

Problem 213: Result more than twice size of optimal antiderivative.

$$\int (b \cosh(x) + c \sinh(x) - \sqrt{b^2 - c^2})^{3/2} dx$$

Optimal(type 3, 82 leaves, 2 steps):

$$-\frac{8(c \cosh(x) + b \sinh(x)) \sqrt{b^2 - c^2}}{3\sqrt{b \cosh(x) + c \sinh(x) - \sqrt{b^2 - c^2}}} + \frac{2(c \cosh(x) + b \sinh(x)) \sqrt{b \cosh(x) + c \sinh(x) - \sqrt{b^2 - c^2}}}{3}$$

Result(type 3, 189 leaves):

$$\frac{(2b^2 - 2c^2) \cosh(x)}{\sqrt{-\frac{\sinh(x) b^2 - \sinh(x) c^2 + b^2 - c^2}{\sqrt{b^2 - c^2}}}} + \frac{\sqrt{-\sqrt{b^2 - c^2} \sinh(x)^2 (\sinh(x) + 1)} \arctan\left(\frac{\sqrt{\sqrt{b^2 - c^2} (\sinh(x) + 1) \cosh(x)}}{\sqrt{-\sqrt{b^2 - c^2} \sinh(x)^2 (\sinh(x) + 1)}}\right) (b^2 - c^2)}{\sqrt{\sqrt{b^2 - c^2} (\sinh(x) + 1) \sinh(x)} \sqrt{-\frac{\sinh(x) b^2 - \sinh(x) c^2 + b^2 - c^2}{\sqrt{b^2 - c^2}}}}$$

Problem 214: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(b \cosh(x) + c \sinh(x) - \sqrt{b^2 - c^2})^{3/2}} dx$$

Optimal(type 3, 134 leaves, 4 steps):

$$\frac{\operatorname{arctanh}\left(\frac{(b^2 - c^2)^{1/4} \sinh(x + \operatorname{I arctan}(b, -Ic)) \sqrt{2}}{2\sqrt{-\sqrt{b^2 - c^2} + \cosh(x + \operatorname{I arctan}(b, -Ic)) \sqrt{b^2 - c^2}}}\right) \sqrt{2}}{4(b^2 - c^2)^{3/4}} + \frac{-c \cosh(x) - b \sinh(x)}{2(b \cosh(x) + c \sinh(x) - \sqrt{b^2 - c^2})^{3/2} \sqrt{b^2 - c^2}}$$

Result(type 1, 1 leaves):???

Problem 215: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)}{a + b \cosh(x) + c \sinh(x)} dx$$

Optimal(type 3, 98 leaves, 4 steps):

$$-\frac{cx}{b^2 - c^2} + \frac{b \ln(a + b \cosh(x) + c \sinh(x))}{b^2 - c^2} - \frac{2ac \operatorname{arctanh}\left(\frac{c - (a - b) \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 + c^2}}\right)}{(b^2 - c^2) \sqrt{a^2 - b^2 + c^2}}$$

Result(type 3, 428 leaves):

$$\frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - 2c \tanh\left(\frac{x}{2}\right) - a - b\right) ab}{(b - c)(b + c)(a - b)} - \frac{\ln\left(a \tanh\left(\frac{x}{2}\right)^2 - \tanh\left(\frac{x}{2}\right)^2 b - 2c \tanh\left(\frac{x}{2}\right) - a - b\right) b^2}{(b - c)(b + c)(a - b)}$$

$$- \frac{2 \operatorname{arctan}\left(\frac{2(a - b) \tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) ac}{(b - c)(b + c)\sqrt{-a^2 + b^2 - c^2}} - \frac{2 \operatorname{arctan}\left(\frac{2(a - b) \tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) cb}{(b - c)(b + c)\sqrt{-a^2 + b^2 - c^2}} + \frac{2 \operatorname{arctan}\left(\frac{2(a - b) \tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) cab}{(b - c)(b + c)\sqrt{-a^2 + b^2 - c^2}(a - b)}$$

$$- \frac{2 \operatorname{arctan}\left(\frac{2(a - b) \tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) cb^2}{(b - c)(b + c)\sqrt{-a^2 + b^2 - c^2}(a - b)} - \frac{4 \ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{4b - 4c} - \frac{4 \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{4b + 4c}$$

Problem 217: Result more than twice size of optimal antiderivative.

$$\int \frac{B \cosh(x) + C \sinh(x)}{(a + b \cosh(x) + c \sinh(x))^2} dx$$

Optimal(type 3, 99 leaves, 4 steps):

$$\frac{2(Bb - Cc) \operatorname{arctanh}\left(\frac{c - (a - b) \tanh\left(\frac{x}{2}\right)}{\sqrt{a^2 - b^2 + c^2}}\right)}{(a^2 - b^2 + c^2)^{3/2}} + \frac{-Bc + bC + aC \cosh(x) + aB \sinh(x)}{(a^2 - b^2 + c^2)(a + b \cosh(x) + c \sinh(x))}$$

Result(type 3, 286 leaves):

$$2 \left( \frac{(B a^2 - B a b + B c^2 + C a c - C b c) \tanh\left(\frac{x}{2}\right)}{a^3 - a^2 b - a b^2 + a c^2 + b^3 - c^2 b} - \frac{B c b + C a^2 - C b^2}{a^3 - a^2 b - a b^2 + a c^2 + b^3 - c^2 b} \right) + \frac{2 \arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) B b}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}}$$

$$- \frac{2 \arctan\left(\frac{2(a-b)\tanh\left(\frac{x}{2}\right) - 2c}{2\sqrt{-a^2 + b^2 - c^2}}\right) C c}{(a^2 - b^2 + c^2)\sqrt{-a^2 + b^2 - c^2}}$$

Problem 220: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\cosh(x)^2 + \sinh(x)^2} dx$$

Optimal (type 3, 3 leaves, 2 steps):

$$\arctan(\tanh(x))$$

Result (type 3, 115 leaves):

$$\frac{2\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{-2 + 2\sqrt{2}} - \frac{2 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{-2 + 2\sqrt{2}} - \frac{2\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2 + 2\sqrt{2}} - \frac{2 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2 + 2\sqrt{2}}$$

Problem 221: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\operatorname{sech}(x)^2 - \tanh(x)^2} dx$$

Optimal (type 3, 15 leaves, 4 steps):

$$-x + \operatorname{arctanh}(\sqrt{2} \tanh(x)) \sqrt{2}$$

Result (type 3, 53 leaves):

$$\sqrt{2} \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right) - 2\right)\sqrt{2}}{4}\right) + \sqrt{2} \operatorname{arctanh}\left(\frac{\left(2 \tanh\left(\frac{x}{2}\right) + 2\right)\sqrt{2}}{4}\right) - \ln\left(1 + \tanh\left(\frac{x}{2}\right)\right) + \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)$$

Problem 222: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{1}{(\coth(x)^2 - \operatorname{csch}(x)^2)^2} dx$$

Optimal(type 1, 1 leaves, 2 steps):

$x$

Result(type 3, 7 leaves):

$$2 \operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Problem 223: Result is not expressed in closed-form.

$$\int \frac{1}{a + b \sinh(x) + c \sinh(x)^2} dx$$

Optimal(type 3, 227 leaves, 7 steps):

$$\frac{2c \operatorname{arctan}\left(\frac{\left(2Ic - (Ib + \sqrt{4ac - b^2}) \tanh\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2(a-c)c - Ib\sqrt{4ac - b^2}}}\right) \sqrt{2}}{\sqrt{4ac - b^2} \sqrt{b^2 - 2(a-c)c - Ib\sqrt{4ac - b^2}}} - \frac{2c \operatorname{arctan}\left(\frac{\left(2Ic - Ib \tanh\left(\frac{x}{2}\right) + \sqrt{4ac - b^2} \tanh\left(\frac{x}{2}\right)\right) \sqrt{2}}{2\sqrt{b^2 - 2(a-c)c + Ib\sqrt{4ac - b^2}}}\right) \sqrt{2}}{\sqrt{4ac - b^2} \sqrt{b^2 - 2(a-c)c + Ib\sqrt{4ac - b^2}}}$$

Result(type 7, 73 leaves):

$$\sum_{R=\operatorname{RootOf}(aZ^4 - 2bZ^3 + (-2a + 4c)Z^2 + 2bZ + a)} \frac{(-R^2 + 1) \ln\left(\tanh\left(\frac{x}{2}\right) - R\right)}{2R^3 a - 3R^2 b - 2Ra + 4Rc + b}$$

Problem 224: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \sinh(x)}{b^2 - 2ab \sinh(x) + a^2 \sinh(x)^2} dx$$

Optimal(type 3, 12 leaves, 3 steps):

$$\frac{\cosh(x)}{b - a \sinh(x)}$$

Result(type 3, 35 leaves):

$$-\frac{2 \left( -\frac{a \tanh\left(\frac{x}{2}\right)}{b} + 1 \right)}{\tanh\left(\frac{x}{2}\right)^2 b + 2a \tanh\left(\frac{x}{2}\right) - b}$$

Problem 225: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)}{a + b \cosh(x) + c \cosh(x)^2} dx$$



Optimal(type 3, 190 leaves, 6 steps):

$$\frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left(1 - \frac{b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} + \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left(1 + \frac{b}{\sqrt{-4ac+b^2}}\right)}{\sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

Result(type 3, 1261 leaves):

$$\begin{aligned} & \frac{c \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{2c \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) a}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\ & - \frac{c \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} - \frac{2c \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) a}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\ & + \frac{c \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} + \frac{c \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\ & + \frac{a \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{2 \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) a^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\ & - \frac{3a \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} - \frac{2 \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) a^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\ & + \frac{3a \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} + \frac{a \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \end{aligned}$$

$$\begin{aligned}
& - \frac{b \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{\operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\
& - \frac{\operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} - \frac{b \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}}
\end{aligned}$$

Problem 226: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(x)^2}{a+b \cosh(x)+c \cosh(x)^2} dx$$

Optimal (type 3, 216 leaves, 7 steps):

$$\frac{x}{c} - \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left( b + \frac{2ac-b^2}{\sqrt{-4ac+b^2}} \right)}{c \sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} - \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left( b + \frac{-2ac+b^2}{\sqrt{-4ac+b^2}} \right)}{c \sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

Result (type 3, 1956 leaves):

$$\begin{aligned}
& - \frac{2 \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) a^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{2 \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) a^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\
& + \frac{\operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} - \frac{\operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b^2}{\sqrt{-4ac+b^2} (c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\
& + \frac{a^2 \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{a^2 \operatorname{arctan} \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{b^2 \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{b^2 \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\
& + \frac{a \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{\sqrt{-4ac+b^2}(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} - \frac{a \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{\sqrt{-4ac+b^2}(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\
& - \frac{b \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} + \frac{a \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\
& + \frac{a \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} - \frac{b \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{c} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{c} - \frac{2c \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{\sqrt{-4ac+b^2}(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\
& + \frac{2c \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{\sqrt{-4ac+b^2}(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} - \frac{2ab \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\
& - \frac{2ab \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right)}{c(c+a-b) \sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} - \frac{\operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right)}{c\sqrt{-4ac+b^2}(c+a-b) \sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} b^3
\end{aligned}$$

$$\begin{aligned}
& + \frac{\arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b^3}{c\sqrt{-4ac+b^2}(c+a-b)\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} - \frac{a^2 \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b}{c\sqrt{-4ac+b^2}(c+a-b)\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \\
& + \frac{2a \operatorname{arctanh} \left( \frac{(-a+b-c) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} \right) b^2}{c\sqrt{-4ac+b^2}(c+a-b)\sqrt{(\sqrt{-4ac+b^2}+a-c)(c+a-b)}} + \frac{a^2 \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b}{c\sqrt{-4ac+b^2}(c+a-b)\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \\
& - \frac{2a \arctan \left( \frac{(c+a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}} \right) b^2}{c\sqrt{-4ac+b^2}(c+a-b)\sqrt{(\sqrt{-4ac+b^2}-a+c)(c+a-b)}}
\end{aligned}$$

Problem 227: Result more than twice size of optimal antiderivative.

$$\int \frac{d + e \cosh(x)}{a + b \cosh(x) + c \cosh(x)^2} dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c-\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{-4ac+b^2}}} \right) \left( e + \frac{-eb+2dc}{\sqrt{-4ac+b^2}} \right)}{\sqrt{b-2c-\sqrt{-4ac+b^2}} \sqrt{b+2c-\sqrt{-4ac+b^2}}} + \frac{2 \operatorname{arctanh} \left( \frac{\sqrt{b-2c+\sqrt{-4ac+b^2}} \tanh\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{-4ac+b^2}}} \right) \left( e + \frac{eb-2dc}{\sqrt{-4ac+b^2}} \right)}{\sqrt{b-2c+\sqrt{-4ac+b^2}} \sqrt{b+2c+\sqrt{-4ac+b^2}}}$$

Result (type ?, 2555 leaves): Display of huge result suppressed!

Problem 228: Result more than twice size of optimal antiderivative.

$$\int \frac{\sinh(x)^3}{\cosh(x)^3 + \sinh(x)^3} dx$$

Optimal (type 3, 29 leaves, 6 steps):

$$\frac{x}{2} + \frac{2 \arctan \left( \frac{(1-2 \tanh(x)) \sqrt{3}}{3} \right) \sqrt{3}}{9} + \frac{1}{6(1+\tanh(x))}$$

Result (type 3, 95 leaves):

$$\frac{1}{3 \left(1 + \tanh\left(\frac{x}{2}\right)\right)^2} - \frac{1}{3 \left(1 + \tanh\left(\frac{x}{2}\right)\right)} + \frac{\ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)}{2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right) - 1\right)}{2} + \frac{I\sqrt{3} \ln\left(\tanh\left(\frac{x}{2}\right)^2 + (I\sqrt{3} - 1) \tanh\left(\frac{x}{2}\right) + 1\right)}{9}$$

$$- \frac{I\sqrt{3} \ln\left(\tanh\left(\frac{x}{2}\right)^2 + (-I\sqrt{3} - 1) \tanh\left(\frac{x}{2}\right) + 1\right)}{9}$$

Problem 229: Result more than twice size of optimal antiderivative.

$$\int \frac{x^2 \operatorname{csch}(x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} dx$$

Optimal (type 4, 89 leaves, 8 steps):

$$-\frac{2x^2 \operatorname{arctanh}(e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} - \frac{2x \operatorname{polylog}(2, -e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} + \frac{2x \operatorname{polylog}(2, e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} + \frac{2 \operatorname{polylog}(3, -e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}} - \frac{2 \operatorname{polylog}(3, e^x) \operatorname{sech}(x)}{\sqrt{a \operatorname{sech}(x)^2}}$$

Result (type 4, 208 leaves):

$$-\frac{e^x x^2 \ln(1 + e^x)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)} - \frac{2 e^x x \operatorname{polylog}(2, -e^x)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)} + \frac{2 e^x \operatorname{polylog}(3, -e^x)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)} + \frac{e^x x^2 \ln(-e^x + 1)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)} + \frac{2 e^x x \operatorname{polylog}(2, e^x)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)}$$

$$- \frac{2 e^x \operatorname{polylog}(3, e^x)}{\sqrt{\frac{a e^{2x}}{(e^{2x} + 1)^2}} (e^{2x} + 1)}$$

Problem 232: Unable to integrate problem.

$$\int \frac{x^3}{a + b \cosh(x) \sinh(x)} dx$$

Optimal (type 4, 334 leaves, 13 steps):

$$\frac{x^3 \ln\left(1 + \frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right)}{\sqrt{4a^2 + b^2}} - \frac{x^3 \ln\left(1 + \frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right)}{\sqrt{4a^2 + b^2}} + \frac{3x^2 \operatorname{polylog}\left(2, -\frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right)}{2\sqrt{4a^2 + b^2}} - \frac{3x^2 \operatorname{polylog}\left(2, -\frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right)}{2\sqrt{4a^2 + b^2}}$$

$$- \frac{3x \operatorname{polylog}\left(3, -\frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right)}{2\sqrt{4a^2 + b^2}} + \frac{3x \operatorname{polylog}\left(3, -\frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right)}{2\sqrt{4a^2 + b^2}} + \frac{3 \operatorname{polylog}\left(4, -\frac{b e^{2x}}{2a - \sqrt{4a^2 + b^2}}\right)}{4\sqrt{4a^2 + b^2}}$$

$$-\frac{3 \operatorname{polylog}\left(4, -\frac{b e^{2x}}{2a + \sqrt{4a^2 + b^2}}\right)}{4\sqrt{4a^2 + b^2}}$$

Result(type 8, 16 leaves):

$$\int \frac{x^3}{a + b \cosh(x) \sinh(x)} dx$$

Problem 236: Unable to integrate problem.

$$\int e^{bx+a} \operatorname{csch}(dx+c) dx$$

Optimal(type 5, 46 leaves, 1 step):

$$-\frac{2 e^{bx+dx+a+c} \operatorname{hypergeom}\left(\left[1, \frac{b+d}{2d}\right], \left[\frac{3}{2} + \frac{b}{2d}\right], e^{2dx+2c}\right)}{b+d}$$

Result(type 8, 15 leaves):

$$\int e^{bx+a} \operatorname{csch}(dx+c) dx$$

Problem 237: Unable to integrate problem.

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^n dx$$

Optimal(type 5, 88 leaves, 2 steps):

$$\frac{(1 + e^{2ex+2d})^n F^{bcx+ac} \operatorname{hypergeom}\left(\left[n, \frac{en+bc \ln(F)}{2e}\right], \left[1 + \frac{en+bc \ln(F)}{2e}\right], -e^{2ex+2d}\right) \operatorname{sech}(ex+d)^n}{en+bc \ln(F)}$$

Result(type 8, 20 leaves):

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^n dx$$

Problem 244: Result is not expressed in closed-form.

$$\int e^x \operatorname{sech}(2x)^2 \tanh(2x) dx$$

Optimal(type 3, 92 leaves, 13 steps):

$$-\frac{e^{5x}}{(1+e^{4x})^2} - \frac{e^x}{4(1+e^{4x})} + \frac{\arctan(e^x \sqrt{2} - 1) \sqrt{2}}{16} + \frac{\arctan(1 + e^x \sqrt{2}) \sqrt{2}}{16} - \frac{\ln(1 + e^{2x} - e^x \sqrt{2}) \sqrt{2}}{32} + \frac{\ln(1 + e^{2x} + e^x \sqrt{2}) \sqrt{2}}{32}$$

Result(type 7, 43 leaves):

$$-\frac{e^x(5e^{4x}+1)}{4(1+e^{4x})^2} + 4 \left( \sum_{R=\text{RootOf}(16777216Z^4+1)} \frac{R \ln(e^x + 64R)}{R} \right)$$

Problem 248: Unable to integrate problem.

$$\int e^{dx+c} \cosh(bx+a) \operatorname{csch}(bx+a)^3 dx$$

Optimal(type 5, 103 leaves, 4 steps):

$$\frac{4e^{2a+c+(2b+d)x} \operatorname{hypergeom}\left(\left[2, 1 + \frac{d}{2b}\right], \left[2 + \frac{d}{2b}\right], e^{2bx+2a}\right)}{2b+d} - \frac{8e^{2a+c+(2b+d)x} \operatorname{hypergeom}\left(\left[3, 1 + \frac{d}{2b}\right], \left[2 + \frac{d}{2b}\right], e^{2bx+2a}\right)}{2b+d}$$

Result(type 8, 23 leaves):

$$\int e^{dx+c} \cosh(bx+a) \operatorname{csch}(bx+a)^3 dx$$

Problem 250: Unable to integrate problem.

$$\int e^{dx+c} \cosh(bx+a)^2 \operatorname{csch}(bx+a) dx$$

Optimal(type 5, 94 leaves, 6 steps):

$$-\frac{3e^{-a+c-(b-d)x}}{2(b-d)} + \frac{e^{a+c+(b+d)x}}{2(b+d)} + \frac{2e^{-a+c-(b-d)x} \operatorname{hypergeom}\left(\left[1, \frac{-b+d}{2b}\right], \left[\frac{b+d}{2b}\right], e^{2bx+2a}\right)}{b-d}$$

Result(type 8, 23 leaves):

$$\int e^{dx+c} \cosh(bx+a)^2 \operatorname{csch}(bx+a) dx$$

Problem 251: Unable to integrate problem.

$$\int e^{dx+c} \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 dx$$

Optimal(type 5, 142 leaves, 5 steps):

$$-\frac{2e^{a+c+(b+d)x} \operatorname{hypergeom}\left(\left[1, \frac{b+d}{2b}\right], \left[\frac{3b+d}{2b}\right], e^{2bx+2a}\right)}{b+d} + \frac{8e^{a+c+(b+d)x} \operatorname{hypergeom}\left(\left[2, \frac{b+d}{2b}\right], \left[\frac{3b+d}{2b}\right], e^{2bx+2a}\right)}{b+d}$$

$$-\frac{8e^{a+c+(b+d)x} \operatorname{hypergeom}\left(\left[3, \frac{b+d}{2b}\right], \left[\frac{3b+d}{2b}\right], e^{2bx+2a}\right)}{b+d}$$

Result(type 8, 25 leaves):

$$\int e^{dx+c} \cosh(bx+a)^2 \operatorname{csch}(bx+a)^3 dx$$

Problem 253: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2}{1 + \tanh(x)^2} dx$$

Optimal(type 3, 3 leaves, 2 steps):

$$\arctan(\tanh(x))$$

Result(type 3, 115 leaves):

$$\frac{2\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{-2 + 2\sqrt{2}} - \frac{2 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{-2 + 2\sqrt{2}}\right)}{-2 + 2\sqrt{2}} - \frac{2\sqrt{2} \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2 + 2\sqrt{2}} - \frac{2 \arctan\left(\frac{2 \tanh\left(\frac{x}{2}\right)}{2 + 2\sqrt{2}}\right)}{2 + 2\sqrt{2}}$$

Problem 254: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2}{2 + 2 \tanh(x) + \tanh(x)^2} dx$$

Optimal(type 3, 5 leaves, 3 steps):

$$\arctan(1 + \tanh(x))$$

Result(type 3, 41 leaves):

$$\frac{I \ln\left(\tanh\left(\frac{x}{2}\right)^2 + (1 - I) \tanh\left(\frac{x}{2}\right) + 1\right)}{2} - \frac{I \ln\left(\tanh\left(\frac{x}{2}\right)^2 + (1 + I) \tanh\left(\frac{x}{2}\right) + 1\right)}{2}$$

Problem 255: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2}{11 - 5 \tanh(x) + 5 \tanh(x)^2} dx$$

Optimal(type 3, 17 leaves, 3 steps):

$$-\frac{2 \arctan\left(\frac{\sqrt{195} (1 - 2 \tanh(x))}{39}\right) \sqrt{195}}{195}$$

Result(type 3, 61 leaves):

$$\frac{I\sqrt{195} \ln\left(11 \tanh\left(\frac{x}{2}\right)^2 + (-I\sqrt{195} - 5) \tanh\left(\frac{x}{2}\right) + 11\right)}{195} - \frac{I\sqrt{195} \ln\left(11 \tanh\left(\frac{x}{2}\right)^2 + (I\sqrt{195} - 5) \tanh\left(\frac{x}{2}\right) + 11\right)}{195}$$

Problem 256: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2 (a + b \tanh(x))}{c + d \tanh(x)} dx$$



Optimal(type 3, 28 leaves, 3 steps):

$$-\frac{(-da+cb)\ln(c+d\tanh(x))}{d^2} + \frac{b\tanh(x)}{d}$$

Result(type 3, 99 leaves):

$$\begin{aligned} & \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2\tanh\left(\frac{x}{2}\right)d + c\right)a}{d} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2\tanh\left(\frac{x}{2}\right)d + c\right)cb}{d^2} + \frac{2\tanh\left(\frac{x}{2}\right)b}{d\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)a}{d} \\ & + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)cb}{d^2} \end{aligned}$$

Problem 257: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2 (a+b\tanh(x))^2}{c+d\tanh(x)} dx$$

Optimal(type 3, 51 leaves, 3 steps):

$$\frac{(-da+cb)^2\ln(c+d\tanh(x))}{d^3} - \frac{b(-da+cb)\tanh(x)}{d^2} + \frac{(a+b\tanh(x))^2}{2d}$$

Result(type 3, 250 leaves):

$$\begin{aligned} & \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2\tanh\left(\frac{x}{2}\right)d + c\right)a^2}{d} - \frac{2\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2\tanh\left(\frac{x}{2}\right)d + c\right)acb}{d^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2\tanh\left(\frac{x}{2}\right)d + c\right)b^2c^2}{d^3} \\ & + \frac{4\tanh\left(\frac{x}{2}\right)^3 ab}{d\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} - \frac{2\tanh\left(\frac{x}{2}\right)^3 b^2c}{d^2\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} + \frac{2b^2\tanh\left(\frac{x}{2}\right)^2}{d\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} + \frac{4\tanh\left(\frac{x}{2}\right)ab}{d\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} - \frac{2\tanh\left(\frac{x}{2}\right)b^2c}{d^2\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^2} \\ & - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)a^2}{d} + \frac{2\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)acb}{d^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)b^2c^2}{d^3} \end{aligned}$$

Problem 258: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{sech}(x)^2 (a+b\tanh(x))^3}{c+d\tanh(x)} dx$$

Optimal(type 3, 74 leaves, 3 steps):

$$-\frac{(-da+cb)^3\ln(c+d\tanh(x))}{d^4} + \frac{b(-da+cb)^2\tanh(x)}{d^3} - \frac{(-da+cb)(a+b\tanh(x))^2}{2d^2} + \frac{(a+b\tanh(x))^3}{3d}$$

Result(type 3, 541 leaves):

$$\begin{aligned}
& \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2 \tanh\left(\frac{x}{2}\right) d + c\right) a^3}{d} - \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2 \tanh\left(\frac{x}{2}\right) d + c\right) a^2 b c}{d^2} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2 \tanh\left(\frac{x}{2}\right) d + c\right) a b^2 c^2}{d^3} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 c + 2 \tanh\left(\frac{x}{2}\right) d + c\right) b^3 c^3}{d^4} + \frac{6 \tanh\left(\frac{x}{2}\right)^5 a^2 b}{d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} - \frac{6 \tanh\left(\frac{x}{2}\right)^5 a b^2 c}{d^2 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{2 \tanh\left(\frac{x}{2}\right)^5 b^3 c^2}{d^3 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{6 \tanh\left(\frac{x}{2}\right)^4 a b^2}{d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} \\
& - \frac{2 \tanh\left(\frac{x}{2}\right)^4 b^3 c}{d^2 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{12 \tanh\left(\frac{x}{2}\right)^3 a^2 b}{d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} - \frac{12 \tanh\left(\frac{x}{2}\right)^3 a b^2 c}{d^2 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{4 \tanh\left(\frac{x}{2}\right)^3 b^3 c^2}{d^3 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{8 \tanh\left(\frac{x}{2}\right)^3 b^3}{3 d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} \\
& + \frac{6 \tanh\left(\frac{x}{2}\right)^2 a b^2}{d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} - \frac{2 \tanh\left(\frac{x}{2}\right)^2 b^3 c}{d^2 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{6 \tanh\left(\frac{x}{2}\right) a^2 b}{d \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} - \frac{6 \tanh\left(\frac{x}{2}\right) a b^2 c}{d^2 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} + \frac{2 \tanh\left(\frac{x}{2}\right) b^3 c^2}{d^3 \left(\tanh\left(\frac{x}{2}\right)^2 + 1\right)^3} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a^3}{d} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a^2 b c}{d^2} - \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) a b^2 c^2}{d^3} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 + 1\right) b^3 c^3}{d^4}
\end{aligned}$$

Problem 260: Unable to integrate problem.

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{4 - \operatorname{sech}(x)^2}} dx$$

Optimal(type 3, 8 leaves, 2 steps):

$$\operatorname{arcsinh}\left(\frac{\tanh(x) \sqrt{3}}{3}\right)$$

Result(type 8, 17 leaves):

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{4 - \operatorname{sech}(x)^2}} dx$$

Problem 261: Unable to integrate problem.

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-4 + \tanh(x)^2}} dx$$

Optimal(type 3, 12 leaves, 3 steps):

$$\operatorname{arctanh}\left(\frac{\tanh(x)}{\sqrt{-4 + \tanh(x)^2}}\right)$$

Result(type 8, 15 leaves):

$$\int \frac{\operatorname{sech}(x)^2}{\sqrt{-4 + \tanh(x)^2}} dx$$

Problem 267: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{coth}(x))^2 \operatorname{csch}(x)^2}{c + d \operatorname{coth}(x)} dx$$

Optimal(type 3, 51 leaves, 3 steps):

$$\frac{b(-da + cb) \operatorname{coth}(x)}{d^2} - \frac{(a + b \operatorname{coth}(x))^2}{2d} - \frac{(-da + cb)^2 \ln(c + d \operatorname{coth}(x))}{d^3}$$

Result(type 3, 202 leaves):

$$\begin{aligned} & -\frac{b^2 \tanh\left(\frac{x}{2}\right)^2}{8d} - \frac{b \tanh\left(\frac{x}{2}\right) a}{d} + \frac{b^2 \tanh\left(\frac{x}{2}\right) c}{2d^2} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) a^2}{d} + \frac{2 \ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) a c b}{d^2} \\ & - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) b^2 c^2}{d^3} - \frac{b^2}{8d \tanh\left(\frac{x}{2}\right)^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) a^2}{d} - \frac{2 \ln\left(\tanh\left(\frac{x}{2}\right)\right) a c b}{d^2} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) b^2 c^2}{d^3} \\ & - \frac{b a}{d \tanh\left(\frac{x}{2}\right)} + \frac{b^2 c}{2d^2 \tanh\left(\frac{x}{2}\right)} \end{aligned}$$

Problem 268: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{coth}(x))^3 \operatorname{csch}(x)^2}{c + d \operatorname{coth}(x)} dx$$

Optimal(type 3, 74 leaves, 3 steps):

$$-\frac{b(-da + cb)^2 \operatorname{coth}(x)}{d^3} + \frac{(-da + cb)(a + b \operatorname{coth}(x))^2}{2d^2} - \frac{(a + b \operatorname{coth}(x))^3}{3d} + \frac{(-da + cb)^3 \ln(c + d \operatorname{coth}(x))}{d^4}$$

Result(type 3, 377 leaves):

$$-\frac{b^3 \tanh\left(\frac{x}{2}\right)^3}{24d} - \frac{3b^2 \tanh\left(\frac{x}{2}\right)^2 a}{8d} + \frac{b^3 \tanh\left(\frac{x}{2}\right)^2 c}{8d^2} - \frac{3b \tanh\left(\frac{x}{2}\right) a^2}{2d} + \frac{3b^2 \tanh\left(\frac{x}{2}\right) a c}{2d^2} - \frac{b^3 \tanh\left(\frac{x}{2}\right) c^2}{2d^3} - \frac{b^3 \tanh\left(\frac{x}{2}\right)}{8d}$$

$$\begin{aligned}
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) a^3}{d} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) a^2 b c}{d^2} - \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) a b^2 c^2}{d^3} \\
& + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2 d + 2c \tanh\left(\frac{x}{2}\right) + d\right) b^3 c^3}{d^4} - \frac{b^3}{24 d \tanh\left(\frac{x}{2}\right)^3} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) a^3}{d} - \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)\right) a^2 b c}{d^2} + \frac{3 \ln\left(\tanh\left(\frac{x}{2}\right)\right) a b^2 c^2}{d^3} \\
& - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right) b^3 c^3}{d^4} - \frac{3 b a^2}{2 d \tanh\left(\frac{x}{2}\right)} + \frac{3 b^2 a c}{2 d^2 \tanh\left(\frac{x}{2}\right)} - \frac{b^3 c^2}{2 d^3 \tanh\left(\frac{x}{2}\right)} - \frac{b^3}{8 d \tanh\left(\frac{x}{2}\right)} - \frac{3 b^2 a}{8 d \tanh\left(\frac{x}{2}\right)^2} + \frac{b^3 c}{8 d^2 \tanh\left(\frac{x}{2}\right)^2}
\end{aligned}$$

Problem 273: Result more than twice size of optimal antiderivative.

$$\int \frac{\cosh(bx+a)^3 - \sinh(bx+a)^3}{\cosh(bx+a)^3 + \sinh(bx+a)^3} dx$$

Optimal(type 3, 40 leaves, 5 steps):

$$-\frac{4 \arctan\left(\frac{(1 - 2 \tanh(bx+a)) \sqrt{3}}{3}\right) \sqrt{3}}{9b} - \frac{1}{3b(1 + \tanh(bx+a))}$$

Result(type 3, 119 leaves):

$$\begin{aligned}
& - \frac{2}{3b \left(\tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)^2} + \frac{2}{3b \left(\tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)} + \frac{2I\sqrt{3} \ln\left(\tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + (-I\sqrt{3} - 1) \tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{9b} \\
& - \frac{2I\sqrt{3} \ln\left(\tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + (I\sqrt{3} - 1) \tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)}{9b}
\end{aligned}$$

Problem 275: Result more than twice size of optimal antiderivative.

$$\int \frac{-\operatorname{csch}(bx+a) + \operatorname{sech}(bx+a)}{\operatorname{csch}(bx+a) + \operatorname{sech}(bx+a)} dx$$

Optimal(type 3, 14 leaves, 2 steps):

$$\frac{1}{b(1 + \tanh(bx+a))}$$

Result(type 3, 35 leaves):

$$\frac{\frac{2}{\left(\tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1\right)^2} - \frac{2}{\tanh\left(\frac{a}{2} + \frac{bx}{2}\right) + 1}}{b}$$

Problem 276: Result more than twice size of optimal antiderivative.

$$\int \frac{-\operatorname{csch}(bx+a)^4 + \operatorname{sech}(bx+a)^4}{\operatorname{csch}(bx+a)^4 + \operatorname{sech}(bx+a)^4} dx$$

Optimal (type 3, 43 leaves, 6 steps):

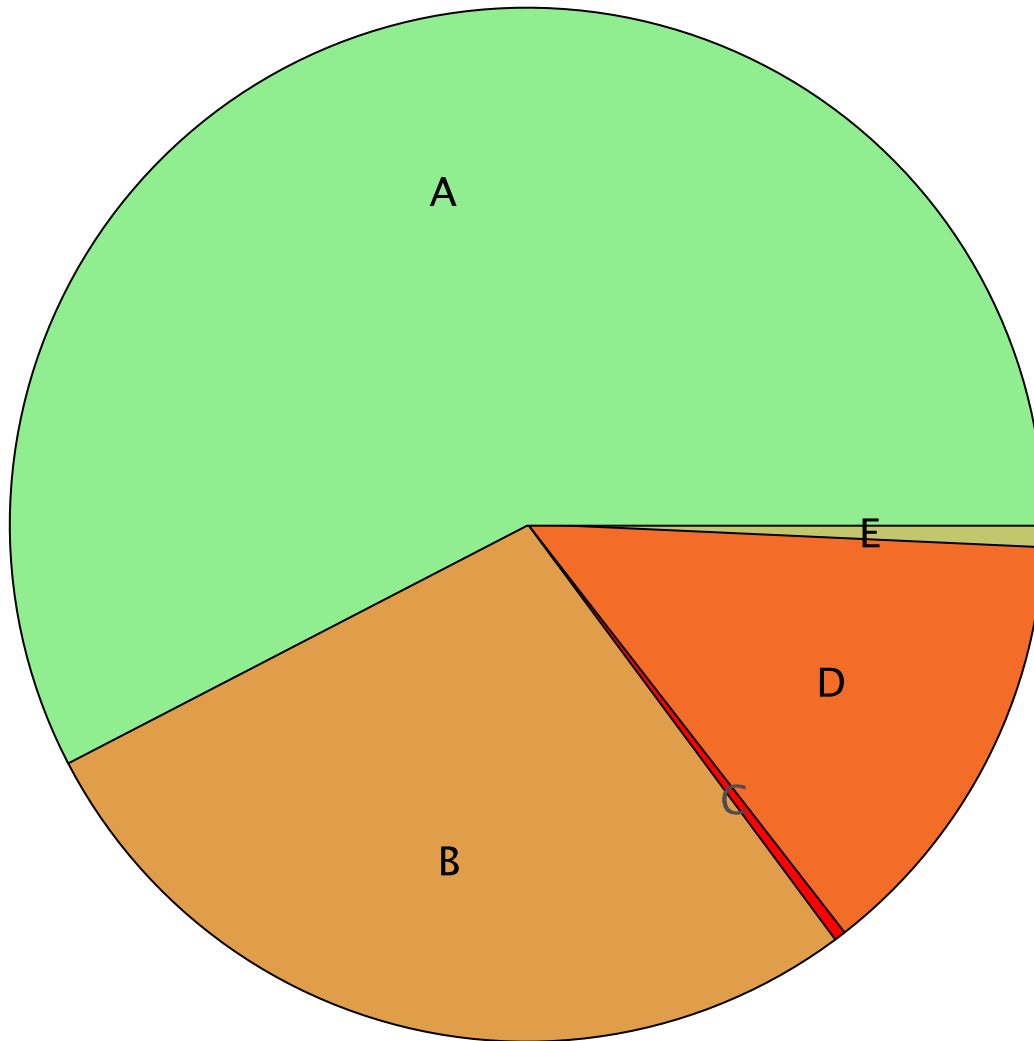
$$-\frac{\arctan\left(\sqrt{2} \tanh(bx+a) - 1\right) \sqrt{2}}{2b} - \frac{\arctan\left(1 + \sqrt{2} \tanh(bx+a)\right) \sqrt{2}}{2b}$$

Result (type 3, 137 leaves):

$$\frac{I\sqrt{2} \ln\left(2I\sqrt{2} \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^3 + \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^4 + 2I\sqrt{2} \tanh\left(\frac{a}{2} + \frac{bx}{2}\right) - 2 \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 1\right)}{4b} - \frac{I\sqrt{2} \ln\left(-2I\sqrt{2} \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^3 + \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^4 - 2I\sqrt{2} \tanh\left(\frac{a}{2} + \frac{bx}{2}\right) - 2 \tanh\left(\frac{a}{2} + \frac{bx}{2}\right)^2 + 1\right)}{4b}$$

Summary of Integration Test Results

276 integration problems



A - 159 optimal antiderivatives  
B - 76 more than twice size of optimal antiderivatives  
C - 1 unnecessarily complex antiderivatives  
D - 38 unable to integrate problems  
E - 2 integration timeouts